Math 111 – 2

Parabolas

A function of the form:

$$f(x) = ax^2 + bx + c, \quad a \neq 0$$

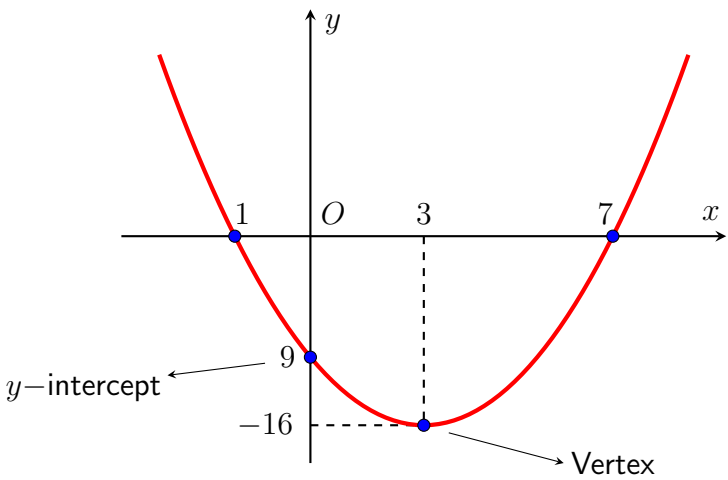
is called a quadratic function. The graph of a quadratic function is a parabola.

- If $a > 0$, the arms of the parabola open upward.
- If $a < 0$, the arms of the parabola open downward.

The vertex of the parabola is maximum or minimum point.

The $x-$coordinate of the vertex is $-\dfrac{b}{2a}$ and the $y-$coordinate is $f\left(-\dfrac{b}{2a}\right)$.

An example is:



The graph of $y = x^2 - 6x - 7$

EXERCISES

Solve the following inequalities:

2–1) $\left|x-7\right| < 8$

2–2) $\left|2x+6\right| \leqslant 4$

2–3) $\left|5x-10\right| > 15$

2–4) $\left|12-7x\right| \geqslant 1$

2–5) $\left|x^2-5\right| < 2$

2–6) $\left|x^2-5\right| < 10$

Sketch the graphs of the following piecewise-defined functions:

2–7) $f(x) = \begin{cases} 2x & \text{if} \quad x < 5 \\ 10 & \text{if} \quad x \geqslant 5 \end{cases}$

2–8) $f(x) = \begin{cases} -2x & \text{if} \quad x < 1 \\ \dfrac{x-7}{3} & \text{if} \quad x \geqslant 1 \end{cases}$

2–9) $f(x) = \begin{cases} x+3 & \text{if} \quad x < 4 \\ x-1 & \text{if} \quad x \geqslant 4 \end{cases}$

2–10) $f(x) = \begin{cases} 2x+5 & \text{if} \quad x < -1 \\ 3 & \text{if} \quad -1 \leqslant x \leqslant 1 \\ -2x+5 & \text{if} \quad x > 1 \end{cases}$

Find the vertex and $x-$ and $y-$ intercepts of the following parabolas. Sketch their graphs:

2–11) $y = x^2 - 6x$

2–12) $y = -x^2 + 3x + 4$

2–13) $y = x^2 + 10x + 25$

2–14) $y = 4x^2 - 8x + 3$

2–15) $y = x^2 - 4x + 5$

ANSWERS

2–1) $-1 < x < 15$

2–2) $-5 \leqslant x \leqslant -1$

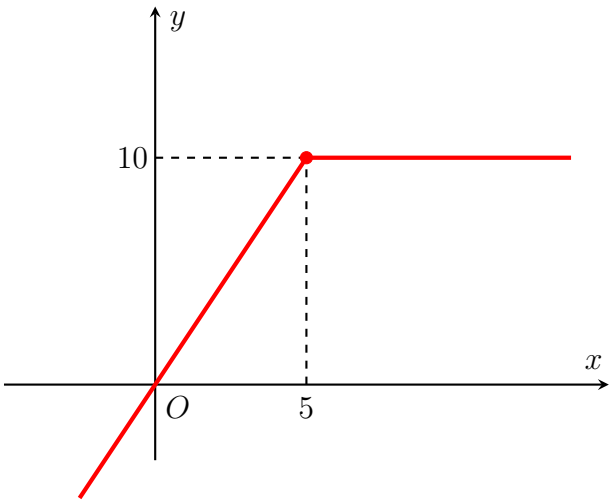
2–3) $x < -1$ or $x > 5$

2–4) $x \leqslant \dfrac{11}{7}$ or $x \geqslant \dfrac{13}{7}$

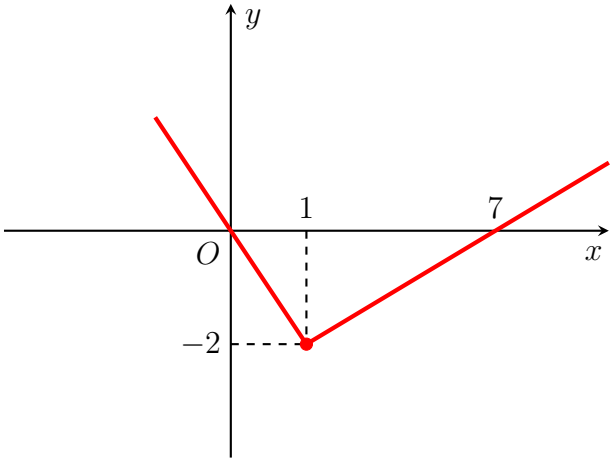
2–5) $\sqrt{3} < x < \sqrt{7}$ or $-\sqrt{7} < x < -\sqrt{3}$

2–6) $-\sqrt{15} < x < \sqrt{15}$

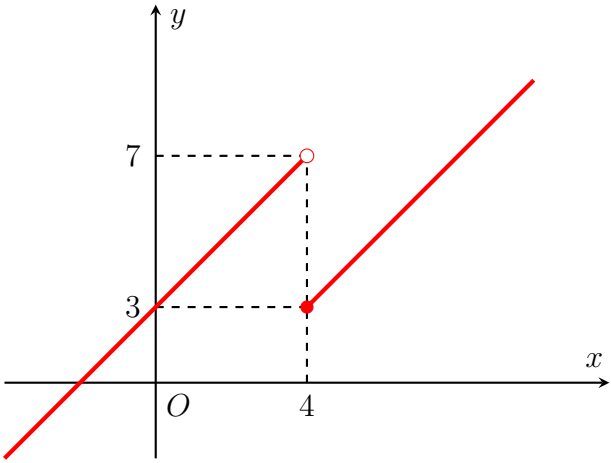
2–7)



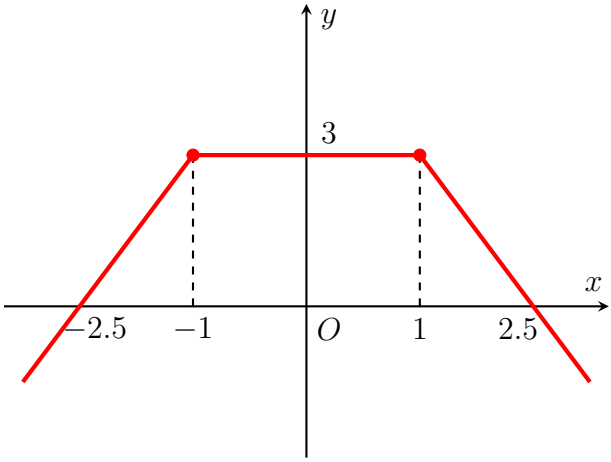
2–8)



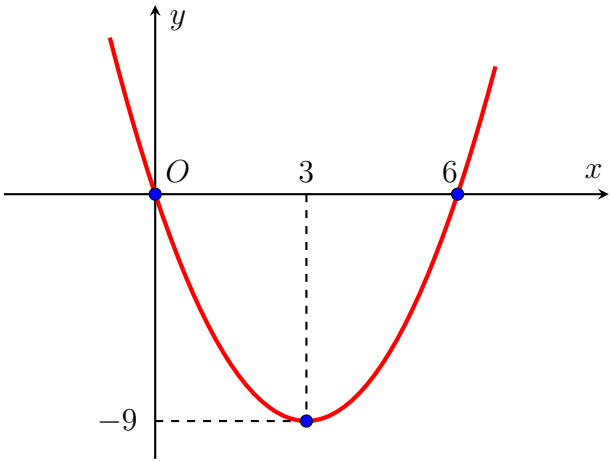
2–9)



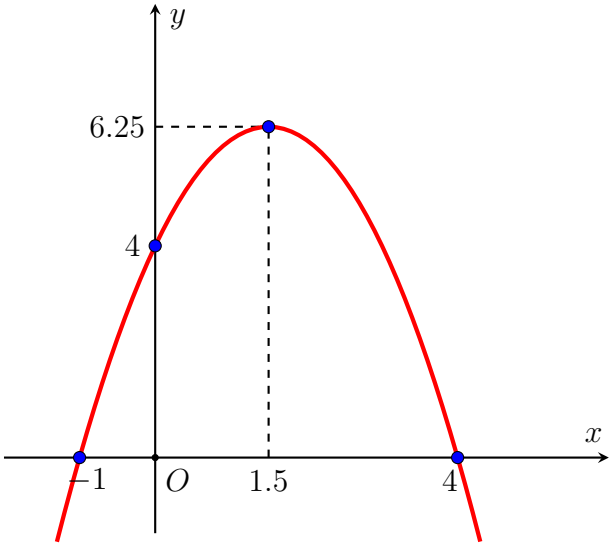
2–10)



2–11)



2–12)



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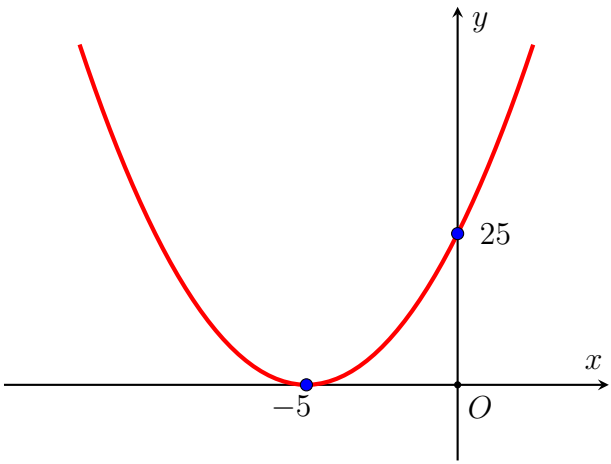

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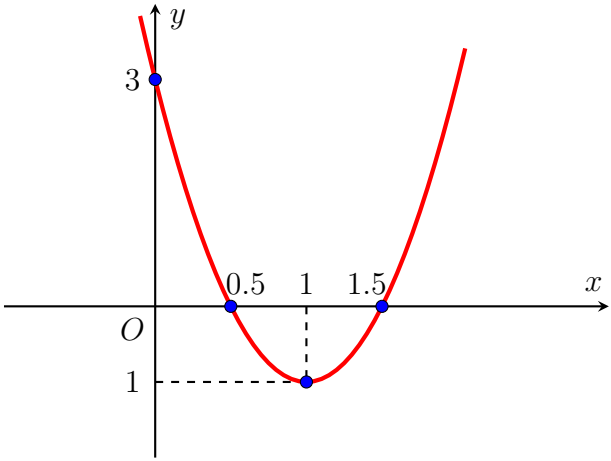

2–15)