KEY

ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MATH 107 - Mathematics For Business and Economics I

MIDTERM 1

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 80 minutes

Question	Grade	Out of
1		15
2		20
3		15
4		15
5		25
6		20
Total		110

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1) Find the solution sets of the following expressions.

a) $\left|\frac{5x-3}{2}\right| > 4$

$\frac{5x-3}{2} > 4$

$5x-3 > 8$

$5x > 11$

$x > \frac{11}{5}$

$\frac{3-5x}{2} > 4$

$3-5x > 8$

$-5 > 5x$

$-1 > x$

$x \in (-\infty, -1) \cup (11/5, \infty)$

b) $\frac{1}{x^2-16} = \frac{1}{x-4} + \frac{1}{x+4}$

$(x+4) \quad (x-4)$

$x \neq 4$

$x \neq -4$

$1 = x+4+x-4$

$1 = 2x$

$x = 1/2$

c) $\sqrt{x^2-x-5} - 2x = 3$

$\left(\sqrt{x^2-x-5}\right)^2 = (3+2x)^2$

$x^2-x-5 = 9+12x+4x^2$

$0 = 3x^2+13x+14$

$0 = (3x+7)(x+2)$

$x = -7/3$
$x = -2$ } None of them satisfies the eqn.

No soln

20 pts.

2) Let $f(x) = \dfrac{1}{x-2}$ and $g(x) = \sqrt{x-1}$.

a) Find $(f-g)(4)$

$$(f-g)(x) = \frac{1}{x-2} - \sqrt{x-1}$$

2 $\quad (f-g)(4) = \dfrac{1}{2} - \sqrt{3}$

b) Find $(fog)(x)$

$$f(\sqrt{x-1}) = \frac{1}{\sqrt{x-1}-2}$$

3

c) Find $(gof)(x)$

3 $\quad g\left(\dfrac{1}{x-2}\right) = \sqrt{\dfrac{1}{x-2} - 1}$

d) Find Dom(g)

4

$\sqrt{x-1} \geq 0$

$x-1 \geq 0$

$x \geq 1$

$\text{Dom}(g) = [1, \infty)$

e) Find Dom$(fog)(x)$

$(fog)(x) = \dfrac{1}{\sqrt{x-1}-2}$

$\sqrt{x-1} - 2 \neq 0$ and $x-1 \geq 0$

$\sqrt{x-1} \neq 2$ $\quad x \geq 1$

$x-1 \neq 4$

$\boxed{x \neq 5}$

4

$\text{Dom}(fog) = [1, \infty) - \{5\}$

e) Find Dom$(gof)(x)$.

$(gof)(x) = \sqrt{\dfrac{1}{x-2} - 1}$

$\dfrac{3-x}{x-2} \geq 0$ and $x \neq 2$

4

	2		3	
$-$		$+$		$-$

$\text{Dom}(gof) = (2, 3]$

3) For the function $f(x) = 2x^2 - 6x - 20$,

a) Find vertex, x-intercept and y-intercept points.
b) Find $Domain(f)$ and $Range(f)$.
c) Sketch the graph of the function.

a) $\left(\frac{6}{4}, f\left(\frac{3}{2}\right)\right)$ = vertex

$\left(\frac{3}{2}, -\frac{49}{2}\right)$

x-int. ; $y=0$ $\quad 0 = 2x^2 - 6x - 20$ $\quad x = 5$

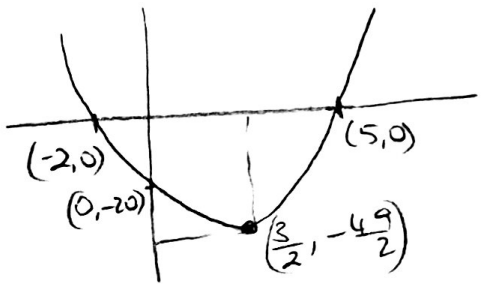
$0 = 2(x-5)(x+2)$ $\quad x = -2$

y-int ; $x = 0$ $\quad f(0) = -20$

b) Domain : $\mathbb{R}$

Range : $\left[-\frac{49}{2}, \infty\right)$

c)



15 pts

4) a) Find equation of a line passing through the point $(-1, 2)$ and perpendicular to the line $2x - 2y + 1 = 0$.

$y = \frac{2x+1}{2}$ $\quad m_1 = 1$

8

Since perpendicular $m_1 \cdot m_2 = -1$ $\quad m_2 = -1$

$y - y_1 = m(x - x_1)$

$y - 2 = -1(x + 1)$

$y = -x + 1$

b) What interest rate is required for \$600 to amount to \$900 in 10 years compounded quarterly?

$S = P\left(1 + \frac{r}{n}\right)^{nt}$

1

$S = 900$

$P = 600$

$n = 4$

$t = 10$

$r = ?$

$900 = 600\left(1 + \frac{r}{4}\right)^{40}$

$\frac{3}{2} = \left(1 + \frac{r}{4}\right)^{40}$

$\sqrt[40]{\frac{3}{2}} = 1 + \frac{r}{4}$

$4\left(\sqrt[40]{\frac{3}{2}} - 1\right) = r$

5) Solve the following equalities.

a) $5(e^{x+1} - 1) = 2$

$$e^{x+1} - 1 = \frac{2}{5}$$

$$e^{x+1} = \frac{7}{5}$$

$$\ln e^{x+1} = \ln 7/5$$

$$(x+1)\cancel{\ln e}^{1} = \ln 7/5$$

$$x = \ln 7/5 - 1$$

b) $\log_2(\frac{x^2+x+5}{16}) = \log_2(4-x) - 4$

$$\log_2\left(\frac{x^2+x+5}{16}\right) - \log_2(4-x) = -4$$

$$\log_2\left(\frac{x^2+x+5}{16(4-x)}\right) = -4$$

$$\frac{x^2+x-5}{16(4-x)} = 2^{-4}$$

$$x^2+x+5 = 4-x$$

$$x^2+2x+1 = 0 \qquad (x+1)^2 = 0 \qquad \boxed{x=-1}$$

c) $\ln(x+6) - \ln(x-2) = \ln(x+1)$

$$\ln(x+6) = \ln(x+1) + \ln(x-2)$$

$$\ln(x+6) = \ln(x+1)(x-2)$$

$$x+6 = (x+1)(x-2)$$

$$x+6 = x^2 - x - 2$$

$$0 = x^2 - 2x - 8$$

$$0 = (x-4)(x+2)$$

$\boxed{x=4}$ soln.

$x=-2 \rightarrow$ doesn't satisfy the eqn.

6) a)Evaluate the following limits.

i) $\lim_{x\to 1} \frac{2x^2-3x+1}{x^2-4x+3}$

$$\lim_{x\to 1} \frac{(2x-1)\cancel{(x-1)}}{\cancel{(x-1)}(x-3)} = \lim_{x\to 1} \frac{2x-1}{x-3} = \frac{1}{-2}$$

ii) $\lim_{x\to\infty} \frac{x^2-1}{4-2x^2}$

$$\lim_{x\to\infty} \frac{\cancel{x^2}}{-2\cancel{x^2}} = \lim_{x\to\infty} \frac{1}{-2} = -1/2$$

b)Let $f(x) = \begin{cases} e^{x^2+2x}, & \text{if } x \geq 0 \\ \frac{x}{x^2+x}, & \text{if } x < 0. \end{cases}$

Is $f(x)$ continuous at $x = 0$? Verify your answer.

1 $f(0) = e^0 = 1$

2 $\lim_{x\to 0^+} e^{x^2+2x} = e^0 = 1$

$\lim_{x\to 0^-} \frac{x}{x^2+x} = \lim_{x\to 0^-} \frac{1}{x+1} = 1$

Equal $\lim_{x\to 0} f(x) = 1$.

3 $f(0) = \lim_{x\to 0} f(x) = 1$

It's cont at $x = 0$