KEY

ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MATH 107 - Mathematics For Business and Economics I

MIDTERM 2

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 75 minutes

Question	Grade	Out of
1		18
2		18
3		20
4		20
5		20
6		14
Total		110

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

18

1) Find the derivatives of the following functions.

4 a) $y = (x^2 - 3x + 5)(x + 2)$

$$y' = (2x-3)(x+2) + (x^2-3x+5).1$$

4 b) $y = \dfrac{x^3 - 3x}{x^2 - 2x + 1}$

$$y' = \frac{(3x^2-3)(x^2-2x+1) - (x^3-3x)(2x-2)}{(x^2-2x+1)^2}$$

5 c) $y = \sqrt[3]{x^3 - 5x - 2}$

$$y' = \tfrac{1}{3}(x^3-5x-2)^{-2/3}(3x^2-5)$$

d) $y = (x^2 - 6x)^3 \cdot (3x + 2)$

5

$$y' = 3(x^2-6x)^2 . (2x-6)(3x+2) + (x^2-6x)^3 (3)$$

18

2) Find the derivatives of the following functions.

a) $y = \ln(5x^2 - 2x)$

3

$$y' = \frac{1}{5x^2-2x} \cdot (10x-2)$$

b) $\sqrt{x} \cdot e^{x^2-3x}$

4

$$y' = \frac{1}{2} x^{-1/2} \cdot e^{x^2-3x} + \sqrt{x} \cdot e^{x^2-3x} \cdot (2x-3)$$

c) $y = x \cdot 3^{x^2-3}$

5

$$y' = 1 \cdot 3^{x^2-3} + x \cdot 3^{x^2-3} \cdot (2x) \cdot \ln 3$$

d) $y = \dfrac{\log_5(x^2-1)}{2x+1}$

6

$$y' = \frac{\frac{1}{x^2-1} \cdot (2x) \cdot \frac{1}{\ln 5} \cdot (2x+1) - \log_5(x^2-1) \cdot 2}{(2x+1)^2}$$

3) a) Evaluate the derivative of $f(x) = y = (x^3 e^{x^2+1})^{x+1}$.

$$\ln y = \ln (x^3 e^{x^2+1})^{x+1}$$

$$\ln y = x+1 \cdot \ln (x^3 e^{x^2+1})$$

$$\frac{y'}{y} = \underbrace{\ln (x^3 e^{x^2+1}) + (x+1) \frac{1}{x^3 e^{x^2+1}} \cdot (3x^2 e^{x^2+1} + x^3 e^{x^2+1} \cdot 2x)}_{(*)}$$

$$y' = (x^3 e^{x^2+1})^{x+1} \cdot (*)$$

b) If $y^3 - x^2 y + x^4 = y^2 + 2x + 1$ is given, evaluate the derivative $\frac{dy}{dx}$ at the point $(1, 2)$.

$$\frac{d}{dx}(y^3 - x^2 y + x^4) = \frac{d}{dx}(y^2 + 2x + 1)$$

$$3y^2 y' - (2xy + x^2 y') + 4x^3 = 2y \cdot y' + 2$$

At $(1,2)$

$$12y' - (4 + y') + 4 = 4y' + 2$$

$$7y' = 2$$

$$\frac{dy}{dx} = y' = 2/7$$

4) a) Find an equation of the tangent line to the curve $y = x^2 \cdot \sqrt{2x-1}$ at the point $(1, 1)$.

$$y' = 2x\sqrt{2x-1} + x^2 \frac{1}{2}(2x-1)^{-1/2} \cdot 2$$

$$y'(1) = 3$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 3(x-1)$$

$$\boxed{y = 3x - 2}$$

b) The total revenue from the sales of a product is given by $R(x) = \dfrac{e^{x^2-11x+10}}{(x-5)}$. Find the marginal revenue function when $x = 10$.

$$R'(x) = \frac{e^{x^2-11x+10} \cdot (2x-11) \cdot (x-5) - e^{x^2-11x+10} \cdot 1}{(x-5)^2}$$

$$R'(10) = \frac{9 \cdot 5 - 1}{(5)^2} = \frac{44}{25}$$

(20)

5) Draw the graph of $f(x) = x^3 - 3x^2 - 24x - 20$ by showing all significant features. That is, **a)** Find f' and the critical values (if exists).
b) Find f'' and the inflection points (if exists).
c) Draw a table showing the signs of f' and f'' and find the relative extrema or absolute extrema (if exists).
d) Sketch the graph of $f(x)$.

a) $f'(x) = 3x^2 - 6x - 24$

(5) $3(x^2 - 2x - 8) = 0$ $\quad$ $\left.\begin{matrix} x=4 \\ x=-2 \end{matrix}\right\}$ cr values

$3(x-4)(x+2) = 0$

64
-48
-96
-20

b) $f''(x) = 6x - 6$

(4) $6(x-1) = 0$ $\quad$ $x = 1$ } inf.

c)

(7)

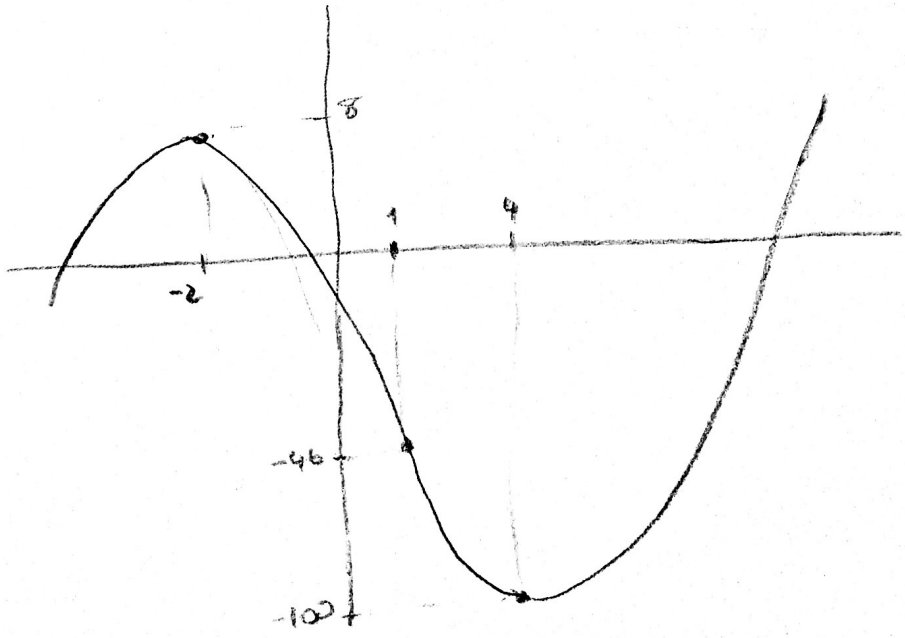
		-2		1		4	
f'	+		−		+		
f''		−		+			
	⌒		⌒		⌣		⌣

$f(4) = -100$
$f(-2) = 8$
$f(1) = -46$

f has a rel. max at $x = -2$ $\quad$ $f(-2) = 8$.
f has a rel. min at $x = 4$ $\quad$ $f(4) = -100$.
No absolute extrema.

d)

(4)



6) Find two numbers such that $3x + y = 9$ and x^2y is a maximum.

$3x+y=9 \Rightarrow y = 9-3x$

$P(x) = x^2(9-3x) = 9x^2 - 3x^3$

$P'(x) = 18x - 9x^2$

$9x(2-x) = 0$

$\left.\begin{array}{l} x=0 \\ x=2 \end{array}\right\}$ cr. values

		0		2	
$P'(x)$	$-$		$+$		$-$

$P(x)$ has a maximum when $x=2$

since $3x+y=9$

$\boxed{y=3}$