ÇANKAYA UNIVERSITY

Department of Mathematics

ANSWERS

MATH 107 - Mathematics For Business and Economics I

5.1.2018

FINAL

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

DURATION: 90 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Total		100

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1) This question has three unrelated parts.

a) You have two investment choices. First choice is that, you may invest your money with 10% compounded quarterly, and the second choice is that, you may invest your money with 10% compounded continuously. Which one is a better choice?

quarterly: $r_e = \left(1 + \frac{0.10}{4}\right)^4 - 1 = 0.1038 = 10.38\ \%$ (5)

continuous: $r_e = e^{0.1} - 1 = 0.1052 = 10.52\ \%$ (5)

continuous is better. (10)

(5) **b)** Find $f'(2)$ where $f(x) = \ln[(x^2 + 3^x)^4]$.

$f(x) = 4 \ln(x^2 + 3^x)$

$f'(x) = \frac{4}{x^2 + 3^x} \cdot (2x + 3^x \ln 3)$ 2

$f'(2) = \frac{4}{13}(4 + 9 \ln 3)$

(5) **c)** Let $xy^2 + x^2y^4 = 16$. Find y'.

$\frac{d}{dx}(xy^2 + x^2y^4) = \frac{d}{dx}(16)$

$y^2 + x\,2yy' + 2xy^4 + x^2\,4y^3y' = 0$

$(2xy + 4x^2y^3)y' = -y^2 - 2xy^4$

$y' = \frac{-y^2 - 2xy^4}{2xy + 4x^2y^3}$

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2) This question has two unrelated parts.

a) Evaluate $\lim_{x\to 2} \dfrac{x^3-8}{x^2-4}$.

$\frac{0}{0} \Rightarrow$ Use L'Hospital

$= \lim_{x\to 2} \dfrac{3x^2}{2x}$ (4)

$= \dfrac{12}{4}$

$= 3$

OR

$\lim_{x\to 2} \dfrac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$

$= \lim_{x\to 2} \dfrac{x^2+2x+4}{x+2}$

$= \dfrac{12}{4}$

$= 3$

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b) Sketch the curve of the equation $f(x) = 2x^3 - 3x^2 - 72x + 10$.

$f' = 6x^2 - 6x - 72$

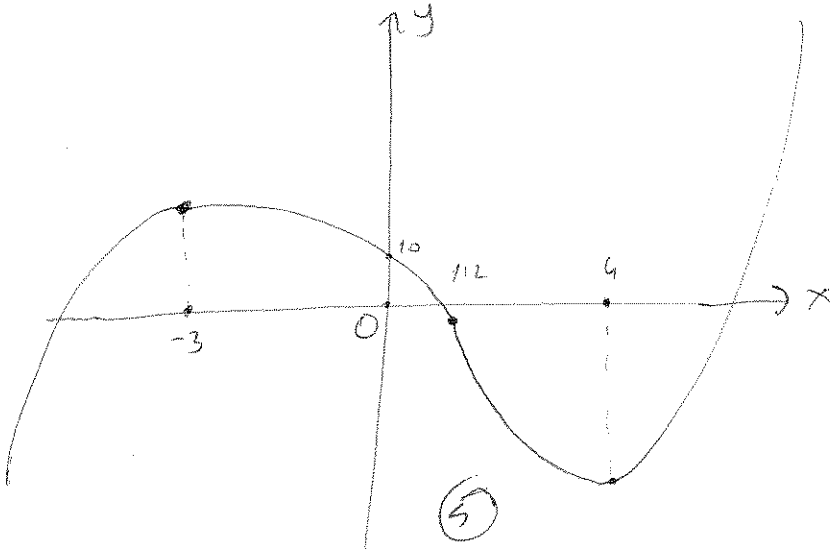
$= 6(x-4)(x+3)$

$f'' = 12x - 6$

$= 6(2x-1)$

$f' = 0 \Rightarrow x = 4$ or $x = -3$ (6)

$f'' = 0 \Rightarrow x = \frac{1}{2}$ (2)



(5)

x		-3		$\frac{1}{2}$		4	
f'	$+$	0	$-$	$-$	0	$+$	
f''	$-$	$-$	0	$+$	$+$	$+$	
f	↗		↘	↘		↗	

3) This question has two unrelated parts.

a) If $2x + 3y = 20$, find the maximum value of x^4y.

$3y = 20 - 2x$

$y = \frac{20-2x}{3}$ 2

$f(x) = x^4 \cdot \left(\frac{20-2x}{3}\right)$

$= \frac{20}{3}x^4 - \frac{2}{3}x^5$

$f'(x) = \frac{80}{3}x^3 - \frac{10}{3}x^4 = 0$

$\frac{10}{3}x^3(8-x) = 0$

$x = 0$ OR $x = 8$ (7)

$f(0) = 0$

$f(8) = 8^3 \cdot \frac{4}{3} = \boxed{\frac{2048}{3}}$

$\boxed{x = 8 \quad y = \frac{4}{3}}$ (3)

b) Find the absolute extrema of the function $f(x) = \frac{x^3}{3} - \frac{5}{2}x^2 + 4x + 6$ on the interval $[2, 5]$.

$f'(x) = x^2 - 5x + 4 = 0$

$= (x-1)(x-4) = 0$

$x = 1$ or $x = 4$ (4)

$1 \notin [2, 5]$

Consider critical point

$x = 4$ and endpoints

$x = 2$ and $x = 5$:

x	$f(x)$
2	$20/3 = 6.67$
(3) 4	$10/3 = 3.33$
5	$31/6 = 5.17$

Abs. Max. is 6.67 $(x = 2)$ (3)

Abs. Min. is 3.33 $(x = 4)$

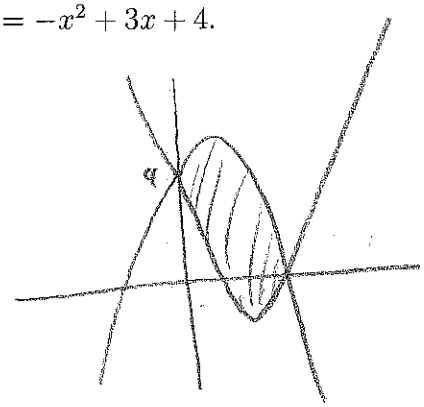
4) This question has two unrelated parts.

a) Evaluate $\int \frac{3x^4 - 2x + x\sqrt{x} + 1}{x} dx$

$= \int \left(3x^3 - 2 + \sqrt{x} + \frac{1}{x}\right) dx$

$= \frac{3x^4}{4} - 2x + \frac{x^{3/2}}{3/2} + \ln|x| + C$

b) Find the area between the curves $y = x^2 - 5x + 4$ and $y = -x^2 + 3x + 4$.



$x^2 - 5x + 4 = -x^2 + 3x + 4$

$2x^2 - 8x = 0$

$2(x-4)x = 0$

$x = 4$

$x = 0$

$$\int_0^4 \left((-x^2+3x+4) - (x^2-5x+4)\right) dx$$

$$= \int_0^4 (-2x^2 + 8x)\, dx$$

$$= -\frac{2x^3}{3} + 4x^2 \Big|_0^4$$

$$= -\frac{128}{3} + 64$$

$$= \frac{64}{3}$$

5) Evaluate the following integrals.

a) $\int_{-1}^{1} (3x^2-1)^7 6x\,dx$

$u = 3x^2-1$

$du = 6x\,dx$

$= \int_{x=-1}^{x=1} u^7\,du$

$= \frac{u^8}{8}\Big|_{x=-1}^{x=1}$

$= \frac{(3x^2-1)^8}{8}\Big|_{-1}^{1} = \frac{2^8}{8} - \frac{2^8}{8} = 0$

b) $\int xe^{-3x}dx = I$

$u = x \qquad dv = e^{-3x}dx$

$du = dx \qquad v = \frac{e^{-3x}}{-3}$

$I = -\frac{xe^{-3x}}{3} - \int \frac{e^{-3x}}{-3}\,dx$

$= -\frac{xe^{-3x}}{3} + \frac{1}{3}\left(\frac{e^{-3x}}{-3}\right) + C$

c) $\int \frac{x+1}{x^2-5x+4}dx = I$

$= (x-4)(x-1)$

$\frac{x+1}{x^2-5x+4} = \frac{A}{x-4} + \frac{B}{x-1}$

$(x-1) \quad (x-4)$

$x+1 = Ax - A + Bx - 4B$

$A+B=1$

$-A-4B=1$ $\Big\}$ $A = 5/3$, $B = -2/3$

$I = \int \left(\frac{5/3}{x-4} + \frac{-2/3}{x-1}\right)dx$

$= \frac{5}{3}\ln|x-4| - \frac{2}{3}\ln|x-1| + C$