KEY

ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 107 - Mathematics For Business and Economics I

30.11.2017

MIDTERM 2

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

DURATION: 90 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Total		100

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1) This question has three unrelated parts.

a) Evaluate $\lim_{x\to\infty} \frac{x^3+4x^2+7}{5-x+3x^3}$.

6

$$\lim_{x\to\infty} \frac{x^3}{3x^3} = \frac{1}{3}$$

6 b) Evaluate $\lim_{x\to 3} \frac{x^2-5x+6}{x^2-9}$.

$$\lim_{x\to 3} \frac{(x-3)(x-2)}{(x-3)(x+3)} = \lim_{x\to 3} \frac{x-2}{x+3}$$

$$= \frac{1}{6}$$

8 c) Let $f(x) = \begin{cases} ax-6 & \text{if } x<2, \\ b & \text{if } x=2, \\ x^2-4 & \text{if } x>2. \end{cases}$

Find the values of a and b, so that the function $f(x)$ is continuous at $x=2$.

$\lim_{x\to 2^+} x^2-4 = 0$ 2

If $f(x)$ is continuous $\lim_{x\to 2^-} ax-6 = \lim_{x\to 2^+} x^2-4 = b$.

Thus, $\lim_{x\to 2^-} ax-6 = 0$

$2a-6=0 \Rightarrow a=3$. 3

And $b=0$. 3

2) Find the derivative of the following functions.

a) $f(x) = (x^2 + 3x)(x^3 + 7x + 9)$

6

$f'(x) = (2x+3)(x^3+7x+9) + (x^2+3x)(3x^2+7)$

7 b) $f(x) = \dfrac{x^4 - 44}{x^2 - 2x}$

$f'(x) = \dfrac{4x^3(x^2-2x) - (x^4-44)(2x-2)}{(x^2-2x)^2}$

7 c) $f(x) = \sqrt[3]{x^3 + 8x + 9}$

$f'(x) = \frac{1}{3}(x^3+8x+9)^{-2/3}(3x^2+8)$

3) a) Find the marginal cost function and evaluate it at $x = 2$ where the cost function is given by

$$C(x) = \frac{8x^4 + 7x^3 + 5x^2}{10x}.$$

$$C(x) = \frac{1}{10}\left(8x^3 + 7x^2 + 5x\right)$$

$$\frac{dC}{dx} = \frac{1}{10}\left(24x^2 + 14x + 5\right)$$

$$\left.\frac{dC}{dx}\right|_{x=2} = \frac{1}{10}\left(24 \cdot 4 + 14 \cdot 2 + 5\right) = \frac{129}{10}$$

b) Find an equation of the tangent line to the graph of the function $y = \frac{8}{x} + x + x\sqrt{x}$ at $x = 4$.

$$y = 8x^{-1} + x + x^{3/2}$$

$$y' = -8x^{-2} + 1 + \frac{3}{2}x^{1/2}$$

$$y'(4) = \frac{-8}{16} + 1 + \frac{3}{2}\sqrt{4}$$

$$= \frac{7}{2}$$

$$y(4) = \frac{8}{4} + 4 + 8 = 14$$

$$y - 14 = \frac{7}{2}(x-4) \Rightarrow \boxed{y = \frac{7}{2}x}$$

4) Find the derivative of the following functions.

a) $f(x) = \ln\left[\left(\frac{x^2+5x+4}{x^3+2}\right)^6\right]$

$$f = 6\left(\ln(x^2+5x+4) - \ln(x^3+2)\right)$$

$$f' = 6\left(\frac{1}{x^2+5x+4}\cdot(2x+5) - \frac{1}{x^3+2}\cdot 3x^2\right)$$

$$= \frac{6(2x+5)}{x^2+5x+4} - \frac{3x^2}{x^3+2}$$

b) $f(x) = xe^{-x^2} + 2^{x^2+x} + \log_4\sqrt{1-x}$

$$f' = e^{-x^2} - 2x^2e^{-x^2} + 2^{x^2+x}(2x+1)\ln 2$$

$$+ \frac{1}{2}\cdot\frac{1}{1-x}\cdot\frac{(-1)}{\ln 4}$$

5) a) Let the equation

$$x^5y^6 + e^{xy} = 3$$

be given. Use implicit differentiation to find $\frac{dy}{dx}$.

$$5x^4y^6 + 6x^5y^5y' + e^{xy}(y + xy') = 0$$

$$y'(6x^5y^5 + xe^{xy}) = -5x^4y^6 - ye^{xy}$$

$$y' = \frac{-5x^4y^6 - ye^{xy}}{6x^5y^5 + x \cdot e^{xy}}$$

b) Let the function

$$f(x) = \frac{(1+x^3)^7(2+x^5)^8}{(3+x^2)^4}$$

be given. Find derivative of the function $f(x)$ at $x = 0$ (in other words, find $f'(0)$).

$$\ln f = 7\ln(1+x^3) + 8\ln(2+x^5) - 4\ln(3+x^2)$$

$$\frac{f'}{f} = \frac{7}{1+x^3} \cdot 3x^2 + \frac{8}{2+x^5} \cdot 5x^4 - \frac{4}{3+x^2} \cdot 2x$$

$$\frac{f'(0)}{f(0)} = 0 \Rightarrow f'(0) = 0$$