10-25

$$\int \frac{1}{x+4}\,dx = \ln|x+4| + C$$

10-33

$$\int \frac{3}{(2-x)^2}\,dx = \int \frac{3}{u^2}(-du) = -3\int u^{-2}\,du = -3\left(\frac{u^{-1}}{-1}\right) + C = \frac{3}{u} + C = \boxed{\frac{3}{2-x} + C}$$

$$\boxed{u = 2-x}$$

$$du = -dx$$

$$\boxed{-du = dx}$$

10-27

$$\int \frac{2x^3+3x}{x^4+3x^2+1}\,dx$$

$$\boxed{u = x^4+3x^2+1}$$

$$du = (4x^3+6x)\,dx$$

$$= 2(2x^3+3x)\,dx$$

$$\frac{1}{2}du = (2x^3+3x)\,dx$$

$$= \int \frac{\frac{1}{2}du}{u} = \frac{1}{2}\ln|u| + C$$

$$= \boxed{\frac{1}{2}\ln(x^4+3x^2+1) + C}$$

10-35

$$\int 2y\sqrt{5-2y^2}\,dy = 2\int \sqrt{5-2y^2}\,y\,dy = 2\int \sqrt{u}\left(-\frac{1}{4}du\right)$$

$$\boxed{u = 5-2y^2}$$

$$du = -4y\,dy$$

$$\boxed{-\frac{1}{4}du = y\,dy}$$

$$= -\frac{1}{2}\int u^{1/2}\,du = -\frac{1}{\cancel{2}}\,\frac{u^{3/2}}{\cancel{3/2}} + C = \boxed{-\frac{1}{3}\left(\sqrt{5-2y^2}\right)^3 + C}$$

10-38

$$\int \left(\frac{6z^2+8z+3}{z+1}\right)dz = \int \left[(6z+2) + \frac{1}{z+1}\right]dz$$

$$\begin{array}{l|l} 6z^2+8z+3 & z+1 \\ \hline -6z^2 \mp 6z & 6z+2 \\ \hline \quad 2z+3 & \\ \quad -2z \mp 2 & \\ \hline \qquad 1 & \end{array}$$

$$= 6\frac{z^2}{2} + 2z + \ln|z+1| + C$$

$$= \boxed{3z^2 + 2z + \ln|z+1| + C}$$

10-42

$$\int_2^3 2e^{x^2}\cdot x\,dx = \int_{x=2}^{x=3} e^{x^2}\cdot(2x\,dx) = \int_{u=4}^{u=9} e^u\cdot du$$

$u=x^2 \Rightarrow du=2x\,dx$

$x=2 \Rightarrow u=4$

$x=3 \Rightarrow u=9$

$$= e^u\Big|_{u=4}^{u=9} = \boxed{e^9-e^4}$$

10-43

$$\int_0^1 x^3\sqrt[3]{x^4+1}\,dx = \int_1^2 u^{1/3}\cdot\frac{1}{4}\,du$$

$$u=x^4+1 \Rightarrow \begin{cases} x=0 \Rightarrow u=1 \\ x=1 \Rightarrow u=2 \end{cases}$$

$du=(4x^3)\,dx$

$\frac{1}{4}du = x^3dx$

$$=\frac{1}{4}\,\frac{u^{4/3}}{4/3}\Big|_1^2$$

$$=\frac{3}{16}\left[\left(2^4\right)^{1/3}-\left(1^4\right)^{1/3}\right]=\frac{3}{16}\left((16)^{1/3}-1\right)$$

10-48

$$\int_0^{\ln 2}\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)dx$$

$$u=e^x+e^{-x} \Rightarrow \begin{cases} x=0 \Rightarrow u=e^0+e^{-0}=1+1=2 \\ x=\ln 2 \Rightarrow u=e^{\ln 2}+\underbrace{e^{-\ln 2}}_{e^{\ln 2^{-1}}}=2+\frac{1}{2}=\frac{5}{2} \end{cases}$$

$2^{-1}=\frac{1}{2}$

$du=(e^x-e^{-x})\,dx$

$$=\int_2^{5/2}\frac{du}{u}=\ln|u|\Big|_2^{5/2}=\ln\frac{5}{2}-\ln 2=\ln\frac{\frac{5}{2}}{2}$$

$$=\ln\frac{5}{4}$$

10-50

$$\int_1^{e^5}\left(\frac{1+2\ln x}{x}\right)dx = \int_1^{11} u\cdot\frac{1}{2}\,du = \frac{1}{2}\,\frac{u^2}{2}\Big|_1^{11}$$

$$=\frac{1}{4}\left(11^2-1^2\right)=\frac{121-1}{4}=\boxed{30}$$

$u=1+2\ln x$: $x=1 \Rightarrow u=1+2\ln 1=1$; $x=e^5 \Rightarrow u=1+2\underbrace{\ln e^5}_{5}=11$

$$du=2\cdot\frac{1}{x}dx \Rightarrow \boxed{\frac{1}{2}du=\frac{dx}{x}}$$

10-56

I.V.P. $\begin{cases} \dfrac{dy}{dx} = \dfrac{1}{2}(1+x)^{-1/2} + 1 \\ y(3) = 5 \end{cases}$ Find the soln. $\Rightarrow$ find $y(x)$

$$\boxed{y(x)} = \int \left[\frac{1}{2}(1+x)^{-1/2} + 1\right] dx = \int \frac{dx}{2\sqrt{(1+x)}} + \int 1\, dx$$

$$\left.\begin{aligned} u &= 1+x \\ du &= dx \end{aligned}\right\} = \frac{1}{2}\int u^{-1/2}\, du + x + C$$

$$= \frac{1}{2}\,\frac{u^{1/2}}{1/2} + x + C = \boxed{\sqrt{1+x} + x + C}$$

$$y(3) = 5 \Rightarrow 5 = \sqrt{1+3} + 3 + C = 2 + 3 + C$$

$$5 = 5 + C \Rightarrow \boxed{C = 0}$$

Soln: $y(x) = \sqrt{1+x} + x$

10-59 $\dfrac{dy}{dx} = -6e^{-2x} - e^{-x}, \; y(0) = 5$

$$y(x) = \int \left(-6e^{-2x} - e^{-x}\right) dx$$

$$= -6\,\frac{e^{-2x}}{-2} - \frac{e^{-x}}{-1} + C$$

$$= \boxed{3e^{-2x} + e^{-x} + C}$$

$$\underset{\overset{\|}{5}}{y(0)} = 3e^{0} + e^{0} + C = \boxed{4 + C}$$

$$\Rightarrow C = 1$$

$$\therefore \; y(x) = 3e^{-2x} + e^{-x} + 1$$

10-60 $\dfrac{dy}{dx} = \dfrac{2x}{1+x^2}, \; y(0) = 0$

$$\boxed{y(x)} = \int \frac{2x}{1+x^2}\, dx = \int \frac{du}{u} = \ln|u| + C = \boxed{\ln(1+x^2) + C}$$

$u = 1 + x^2$

$du = (2x)dx$

$$y(0) = 0 \Rightarrow 0 = y(0) = \underbrace{\ln 1}_{0} + C \Rightarrow \boxed{C = 0}$$

$\therefore$ Soln: $y(x) = \ln(1+x^2)$