ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 111 - Mathematics for Economics and Social Sciences I

2022-2023 Fall

FIRST MIDTERM EXAMINATION

17.11.2022, 17:30

- SOLUTIONS -

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

DURATION: 90 minutes

Question	Grade	Out of
1		20
2		27
3		25
4		27
5		12
Total		111

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) **SHOW ALL YOUR WORK**. No points will be given to correct answers without reasonable work.

1. Find the solution sets of the following expressions. Clearly indicate the solution sets.

a) (10 points) $\sqrt{2x+7} = x-4$

$\left(\sqrt{2x+7}\right)^2 = (x-4)^2 \Rightarrow 2x+7 = x^2-8x+16 \Rightarrow x^2-10x+9=0$

$(x-9)(x-1)=0$

$\boxed{x=9,\ x=1}$

Check in $\sqrt{2x+7} = x-4$:

$x_1=9 \Rightarrow \sqrt{2(9)+7}=\sqrt{25}=5=9-4$ ✓ $\Rightarrow x_1=9$ satisfies the original eqn.

$x_2=1 \Rightarrow \sqrt{2(1)+7}=3 \neq 1-4=-3$ ✗ $\Rightarrow x_2=1$ does not satisfy the original eqn.

$\Rightarrow$ Solution set: $\boxed{\{9\}}$

b) (10 points) $|3x-2|+x > \frac{5}{2}$

$|3x-2| > \frac{5}{2}-x \Rightarrow 3x-2 > \frac{5}{2}-x \Rightarrow 4x > \frac{5}{2}+2 = \frac{9}{2} \Rightarrow \boxed{x > \frac{9}{8}}$

or $-(3x-2) > \frac{5}{2}-x \Rightarrow -3x+2 > \frac{5}{2}-x \Rightarrow -2x > \frac{1}{2}$

$\Rightarrow \boxed{x < -\frac{1}{4}}$

$\Rightarrow \quad x > \frac{9}{8} \Rightarrow \left(\frac{9}{8}, \infty\right)$

$x < -\frac{1}{4} \Rightarrow \left(-\infty, -\frac{1}{4}\right)$

$\Rightarrow$ Solution set: $\boxed{\left(-\infty, -\frac{1}{4}\right) \cup \left(\frac{9}{8}, \infty\right)}$

2. a) (7 points) Write the following expression as a single logarithm in the most simplified form:

$\frac{1}{3}\ln x + 3\ln(x^2) - 2\ln(x-1) - 3\ln(x-2) = ?$

$= \ln x^{\frac{1}{3}} + \ln(x^2)^3 - \ln(x-1)^2 - \ln(x-2)^3$

$= \ln\left[\frac{x^{1/3}\cdot x^6}{(x-1)^2(x-2)^3}\right] = \boxed{\ln\left[\frac{x^{19/3}}{(x-1)^2(x-2)^3}\right]}$

b) (6 points) Find the solution set of: $3^{4x} = 9^{x+1}$.

$3^{4x} = (3^2)^{x+1} = 3^{2x+2} \iff 4x = 2x+2$

$\Rightarrow 2x = 2$

$\Rightarrow \boxed{x=1}$

Soln. set: $\{1\}$

c) (7 points) Find the solution set of: $\log(6x^2 - x - 11) = 0$.

$\log \to 10$

$\Rightarrow 10^0 = 6x^2 - x - 11 \Rightarrow 6x^2 - x - 11 - 1 = 0$

$10^0 = 1$

$6x^2 - x - 12 = 0$

$(3x+4)(2x-3) = 0$

$\Rightarrow \boxed{x = -\frac{4}{3} \text{ or } x = \frac{3}{2}}$

Soln. set: $\left\{-\frac{4}{3}, \frac{3}{2}\right\}$

d) (7 points) Find the solution set of: $\log_x(6 - 4x - x^2) = 2$.

$\log_x \to x > 0,\ x \neq 1$

$\log_x(6-4x-x^2) = 2 \iff x^2 = 6 - 4x - x^2 \Rightarrow 2x^2 + 4x - 6 = 0$

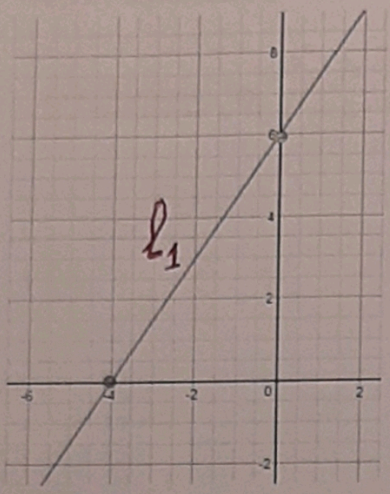
$\Rightarrow 2(x^2 + 2x - 3) = 0$

$(x+3)(x-1) = 0 \Rightarrow x = -3$ (contradicts with $x > 0$)

$x = 1$ (contradicts with $x \neq 1$)

$\Rightarrow$ so there is no solution! $\Rightarrow$ Soln. set: $\{\emptyset\}$

3. a) (6 points) Find the equation of the following line l_1 :



l_1 passes through $(0,6)$ and $(-4,0) \Rightarrow$

slope of $l_1 = m_1 = \frac{6-0}{0-(-4)} = \frac{6}{4} = \frac{3}{2}$

$\Rightarrow y-0 = \left(\frac{3}{2}\right)(x-(-4))$

$\Rightarrow y = \frac{3}{2}x + \left(\frac{3}{2}\right)(4) = \frac{3}{2}x+6$

$\Rightarrow \boxed{y = \frac{3}{2}x+6}$ or $\boxed{2y-3x=12}$

b) (6 points) Find the equation of the line l_2 passing through the point $(6,2)$ and perpendicular to the line l_1 given in part a).

$l_2 \perp l_1 \Rightarrow$ slope of $l_2 = m_2 = \frac{-1}{m_1} = -\frac{1}{\left(\frac{3}{2}\right)} = \boxed{-\frac{2}{3}}$

l_2: has slope $-\frac{2}{3}$ & passes through $(6,2)$ $\Rightarrow y-2 = \left(-\frac{2}{3}\right)(x-6) \Rightarrow 3y-6 = -2x+12$

$\boxed{2x+3y=18}$ (or: $\boxed{y=-\frac{2}{3}x+6}$)

c) (8 points) Find the inverse function $f^{-1}(x)$ given that $f(x) = 5e^{3x-2}$.

(Hint: Change the roles of x & y in $y = 5e^{3x-2}$ and use the fact that $y = e^u \iff \ln y = u$)

$y = 5e^{3x-2} \xrightarrow[x \leftrightarrow y]{} x = 5e^{3y-2} \Rightarrow \frac{x}{5} = e^{3y-2} \Rightarrow \ln\left(\frac{x}{5}\right) = 3y-2$

$\Rightarrow 3y = \ln\left(\frac{x}{5}\right)+2 \Rightarrow \boxed{y = \frac{\ln\left(\frac{x}{5}\right)+2}{3}} \Rightarrow y = f^{-1}(x)$ $\boxed{f^{-1}(x) = \frac{\ln\left(\frac{x}{5}\right)+2}{3}}$

d) (2 points) Find the Vertex of the parabola given by the equation $g(x) = -2(x+3)^2+32$.

$\boxed{V(-3,32)}$

e) (3 points) What is the Range of $g(x)$ given in part d).

$a = -2 < 0 \Rightarrow$ Vertex $\rightarrow$ max. pt

$\boxed{\text{Range } g(x) = (-\infty, 32]}$

4. Evaluate the following limits if they exist. Express the solutions clearly. DO NOT USE L'HOPITAL'S RULE!

a) (9 points) $\lim_{x\to 4} \frac{x^2-7x+10}{x^2-5x+6} = \frac{(4)^2-7(4)+10}{(4)^2-5(4)+6} = \frac{26-28}{22-20}$

$= \frac{-2}{2} = \boxed{-1}$

b) (9 points) $\lim_{x\to 5^+} \frac{x^2-7x+10}{\sqrt{x-5}} = \lim_{x\to 5^+} \frac{(x-5)(x-2)}{\sqrt{x-5}} = \frac{0}{0}$

$= \lim_{\substack{x\to 5^+ \\ (x>5)}} \left(\frac{(\sqrt{x-5})(\sqrt{x-5})(x-2)}{\sqrt{x-5}} \right) = (0)(5-2) = (0)(3)$

$= \boxed{0}$

c) (9 points) $\lim_{x\to\infty} \left(\sqrt{x^2+2x} - \sqrt{x^2-3x+5} \right) = \infty - \infty$

$= \lim_{x\to\infty} \left[\frac{(\sqrt{x^2+2x} - \sqrt{x^2-3x+5})}{\sqrt{x^2+2x} + \sqrt{x^2-3x+5}} \cdot (\sqrt{x^2+2x} + \sqrt{x^2-3x+5}) \right]$

$= \lim_{x\to\infty} \left[\frac{(x^2+2x)-(x^2-3x+5)}{\sqrt{x^2(1+\frac{2}{x})} + \sqrt{x^2(1-\frac{3}{x}+\frac{5}{x^2})}} \right] = \lim_{x\to\infty} \left[\frac{x(5-\frac{5}{x})}{x\left(\sqrt{1+\frac{2}{x}} + \sqrt{1-\frac{3}{x}+\frac{5}{x^2}}\right)} \right]$

$= \frac{5-0}{\sqrt{1+0}+\sqrt{1-0+0}} = \boxed{\frac{5}{2}}$

5. (12 points) Find the values of the constants a and b that will make the following function continuous everywhere:

$$f(x) = \begin{cases} a + 3x^2, & \text{if } x < 0, \\ 3, & \text{if } x = 0, \\ b + e^{-x}, & \text{if } x > 0. \end{cases}$$

Explain your answer in detail.

$f(0) = 3$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (a + 3x^2) = \boxed{a}$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (b + e^{-x}) = b + e^0 = \boxed{b+1}$

$\Rightarrow$

$\Rightarrow \lim_{x \to 0} f(x)$ exists and is $\boxed{a = b+1}$

$\Rightarrow f$ is continuous at $x=0 \Leftrightarrow a = b+1 = 3 = f(0)$

$\Rightarrow \boxed{a=3}$

$b+1=3 \Rightarrow \boxed{b=2}$

At all other pts.: (other than $x=0$)

$\{3+3x^2, 2+e^{-x}\}$ are cont.

So; f is cont. everywhere if $\boxed{\begin{matrix} a=3 \\ b=2 \end{matrix}}$