3-24) $3^{2\log_9 x} = ?$

Method 1: $(a^x)^y = a^{xy}$

$3^{2\log_9 x} = (3^2)^{\log_9 x} = 9^{\log_9 x} = \boxed{x}$

Method 2:

$\log_9 x = \frac{\log_3 x}{\log_3 9} = \frac{\log_3 x}{2\log_3 3} = \frac{1}{2}\log_3 x$ ($\log_3 3 = 1$)

$3^{2\log_9 x} = 3^{\frac{1}{2}\log_3 x}$

$= 3^{\log_3 x} = \boxed{x}$

$\boxed{a^{\log_a x} = x}$

3-25) $5^{\log_{25} x} = ?$

$\log_{25} x = \frac{\log_5 x}{\log_5 25} = 2\log_5 5 = 2$ ($\log_5 5 = 1$)

change-of-base formula

$\rightarrow = \boxed{\frac{1}{2}\log_5 x}$

$5^{\log_{25} x} = 5^{\frac{1}{2}\log_5 x}$

$= 5^{\log_5 x^{1/2}}$

$= 5^{\log_5 \sqrt{x}} = \boxed{\sqrt{x}}$

3-26) $10^{1+\log(2x)} = 10 \cdot 10^{\log(2x)}$ ($\log = \log_{10}$)

$= (10)(2x) = \boxed{20x}$

Solve the following equations:

3-28) $\log_x (12) = \frac{1}{2}$

$\boxed{x>0, x\neq 1}$

$\Rightarrow (x^{1/2})^2 = (12)^2$

$\boxed{x = 144}$

3-31) $\log_x 64 = 4 \Leftrightarrow x^4 = 64 \Rightarrow x^2 = 8$

$\Rightarrow \boxed{x = \mp 2\sqrt{2}}$

$\boxed{x>0, x\neq 1}$

$x = -2\sqrt{2}$ contradicts $x>0$

Soln: $\{2\sqrt{2}\}$

eg: $\log_x (1) = 2 \Leftrightarrow x^2 = 1$

$\Rightarrow \boxed{x = \mp 1}$ ✗

$\Downarrow$

$(x>0, x\neq 1)$

✓ $x=1 \Rightarrow$ contradicts $x\neq 1$

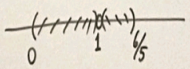
✓ $x=-1 \Rightarrow$ // $x>0$

Soln: No solution (or $\{\emptyset\}$)

eg. $\log_x(6-5x)=2$

i) $x>0,\ x\neq 1$

ii) $6-5x>0 \Rightarrow 5x<6 \Rightarrow x<\frac{6}{5}$



$\hookrightarrow x^2=6-5x$

$\Rightarrow x^2+5x-6=0$

$(x+6)(x-1)=0$

$\Rightarrow x=-6$ contradicts $x>0$

$x=1$ contradicts $x\neq 1$

$\Rightarrow$ No soln. or $(\{\phi\})$

3-33) $\log_9(18x)=2 \Rightarrow 9^2=18x \Rightarrow 81=18x$

$\Rightarrow x=\frac{81}{18}=\boxed{\frac{9}{2}}$

3-35) $\log_{10}(\log x)=0$

$\Rightarrow \underbrace{10^0}_{1}=\log x \Rightarrow$

$\log_{10} x=1 \Rightarrow x=10^1=\boxed{10}$

3-36) $\ln_e(\ln x)=1$

$\Rightarrow e=\ln_e x \Rightarrow \boxed{x=e^e}$

3-38) $\boxed{2^{4x+4}}=8^{x-1}$

$=(2^3)^{x-1}=\boxed{2^{3x-3}}$

$\Leftrightarrow 4x+4=3x-3$

$\boxed{x=-7}$

Simplify & solve:

$\left(3^{2\log_3 x}\right)\left(9^{(\log_4 2)}\right)=6 \Rightarrow (x^2)\left(9^{\frac{1}{2}}\right)=6$

$* 3^{2\log_3 x}=3^{\log_3 x^2}=x^2$

$* \log_4 2=\frac{\log_2 2}{\log_2 2^2}=\frac{1}{2\cdot 1}=\boxed{\frac{1}{2}}$

$x^2\cdot 3=6$

$x^2=2$

$\boxed{x=\mp\sqrt{2}}$

$x=\sqrt{2}$ ✓

$x=-\sqrt{2}$ ✗ contradicts w/ dom $\log x$ being $(0,\infty)$

Soln.:

$\{\sqrt{2}\}$