M1 Contents:

Chs. 1-5 from the Lecture notes

- Functions
- Parabolas
- Exponential & Logarithmic Functions
- Limits
- One-sided limits, Continuity

Ex. 5-12 Let $f(x)$ be given by:

$$f(x)=\begin{cases} \log_{10}\left(\frac{x}{2}+b\right) & \text{if } x<8 \\ x\left(\sqrt{x-8}+\frac{1}{4}\right) & \text{if } x\geq 8 \end{cases}$$

$\boxed{\text{Find } b}$ if $f(x)$ is continuous at $x=8$.

Cont.:

i) Is $f(8)$ defined?:

ii) Does $\lim_{x\to 8} f(x)$ exist? $\lim_{x\to 8} f(x)=?$

iii) $f(8)=\lim_{x\to 8} f(x)$

* $f(8)=8\left(\sqrt{\cancel{8-8}}+\frac{1}{4}\right)=\boxed{2}$ ($8-8=0$)

* $$\lim_{\substack{x\to 8^- \\ (x<8)}} f(x)=\lim_{x\to 8^-}\left[\log_{10}\left(\frac{x}{2}+b\right)\right]$$

$$=\log_{10}\left(\frac{8}{2}+b\right)=\boxed{\log_{10}(4+b)}$$

$$\lim_{\substack{x\to 8^+ \\ (x>8)}} f(x)=\lim_{x\to 8^+}\left[x\left(\sqrt{x-8}+\frac{1}{4}\right)\right]=\boxed{2}$$

For continuity at $x=8$:

$$\log_{10}(4+b)=2$$

$$\Leftrightarrow 10^2=4+b$$

$$100-4=b$$

$$\boxed{b=96}$$

5-22)

$$f(x)=\begin{cases} cx^2-2 & \text{if } x\leq 2 \\ \frac{x}{c} & \text{if } x>2 \end{cases}$$

* $f(2)=c(2)^2-2=\boxed{4c-2}$

* $\lim_{x\to 2} f(x)=?$

$$\lim_{\substack{x\to 2^- \\ (x<2)}} f(x)=\lim_{x\to 2^-}(cx^2-2)=c(2)^2-2=\boxed{4c-2}$$

$$\lim_{\substack{x\to 2^+ \\ (x>2)}} f(x)=\lim_{x\to 2^+}\left(\frac{x}{c}\right)=\boxed{\frac{2}{c}}$$

so we can have a limit at $x=2$

$$\Rightarrow \quad 4c-2=\frac{2}{c} \Rightarrow 4c^2-2c-2=0$$

$$2(2c^2-c-1)=0$$

$$2(2c+1)(c-1)=0$$

$$\boxed{c=-\tfrac{1}{2},\ c=1}$$

$\Rightarrow$

$c=-\frac{1}{2}$:

$\lim_{x\to 2} f(x)=-4=f(2)$

$c=1$:

$\lim_{x\to 2} f(x)=2=f(2)$

$f(x)$ is cont. at $x=2$ for $c=-\frac{1}{2}$ and $c=1$.

5-23) $f(x)=\begin{cases} x^2-c^2, & \text{if } x\le 1 \\ (x-c)^2, & \text{if } x>1 \end{cases}$

Since $f(x)$ is a polynomial for $x<1$ and $x>1 \Rightarrow$ it is continuous for all these x-values.

At $x=1$:

$* \; f(1)=1^2-c^2=\boxed{1-c^2}$

$* \; \lim\limits_{x\to 1} f(x)=?$

$- \lim\limits_{\substack{x\to 1^- \\ (x<1)}} f(x)=\lim\limits_{x\to 1^-}(x^2-c^2)=1-c^2$

$- \lim\limits_{\substack{x\to 1^+ \\ x>1}} f(x)=\lim\limits_{x\to 1^+}(x-c)^2=(1-c)^2=\boxed{1-2c+c^2}$

For $\lim\limits_{x\to 1} f(x)$ to exist we should have:

$1-c^2=1-2c+c^2$

$\cancel{1}-2c+c^2-\cancel{1}+c^2=0$

$2c^2-2c=0$

$2c(c-1)=0 \Rightarrow \boxed{c=0}, \; \boxed{c=1}$

For $c=0$:

$\lim\limits_{x\to 1} f(x)=1=f(1) \Rightarrow f$ is continuous at $x=1$ for $c=0$.

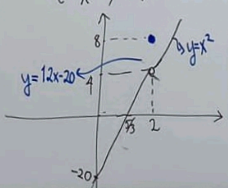
$c=1$:

$\lim\limits_{x\to 1} f(x)=0=f(1) \Rightarrow f$ is continuous at $x=1$ for $c=1$ as well.

$\therefore$ f is continuous everywhere for the values $c=0$ & $c=1$ of the constant "c"

5-20) Find the discontinuities of the following function. Justify your answer

$f(x)=\begin{cases} 12x-20, & \text{if } x<2 \\ 8, & \text{if } x=2 \\ x^2, & \text{if } x>2 \end{cases}$



$* \; f(2)=8$

$* \; \lim\limits_{\substack{x\to 2^- \\ (x<2)}} f(x)=\lim\limits_{\substack{x\to 2^- \\ x<2}}(12x-20)=4=\lim\limits_{\substack{x\to 2^+ \\ (x>2)}} f(x)=\lim\limits_{x\to 2^+}(x^2)=4 \Rightarrow \lim\limits_{x\to 2} f(x)=4$

$*$ f is discontinuous at $x=2$ since $8=f(2)\neq \lim\limits_{x\to 2} f(x)=4$

If $f(2)$ was defined as "a" then choosing $f(2)=a=4=\lim\limits_{x\to 2} f(x)$ we would have f cont. at $x=2$ as well $\Rightarrow$ hence everywhere.