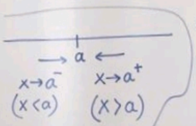
One-sided Limits:

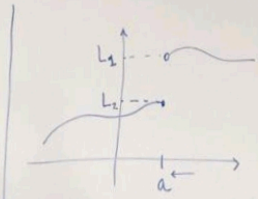
* When x approaches "a" from values larger than "a" we write: $x \to a^+$ $(x > a)$

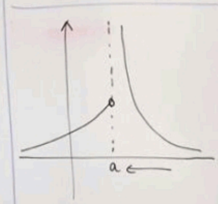


* When x approaches "a" from values less than "a" we write: $x \to a^-$ $(x<a)$

* If $\lim\limits_{x \to a^+} f(x) = L_1 \Rightarrow$ we say that L_1 is the right-limit of f at x=a

* If $\lim\limits_{x \to a^-} f(x) = L_2 \Rightarrow$ we say that L_2 is the left-limit of f at x=a





Theorem: The limit $\lim\limits_{x \to a} f(x) = L \Leftrightarrow$ both of the one-sided limits

$$\boxed{\lim_{x \to a^+} f(x) \quad \& \quad \lim_{x \to a^-} f(x)}$$

exist and are equal.

$$\lim_{x \to a} f(x) = L \Leftrightarrow \lim_{\substack{x \to a^- \\ (x<a)}} f(x) = L = \lim_{\substack{x \to a^+ \\ (x>a)}} f(x)$$

Ex.5.1: Consider the function:

$$f(x) = \begin{cases} 2x-1 & \text{if } x<1 \\ 5x-2 & \text{if } x>1 \end{cases}$$

Find $\lim\limits_{x \to 1^-} f(x)$, $\lim\limits_{x \to 1^+} f(x)$, & $\lim\limits_{x \to 1} f(x)$?

$$\lim_{\substack{x \to 1^- \\ (x<1)}} f(x) = \lim_{x \to 1^-} (2x-1) = 2(1)-1 = \boxed{1}$$

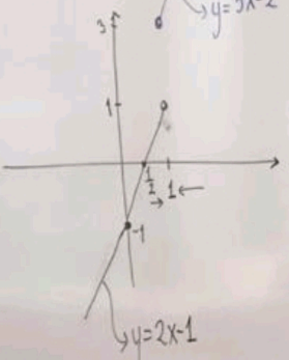
left-limit at x=1

$$\lim_{\substack{x \to 1^+ \\ (x>1)}} f(x) = \lim_{x \to 1^+} (5x-2) = 5(1)-2 = \boxed{3}$$

right-limit at x=1

Since; $1 = \lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x) = 3$

$\Rightarrow \boxed{\lim\limits_{x\to 1} f(x) = \text{d.n/e}}$ (does not exist)



Ex.5.2: $\lim\limits_{x\to 3^+} f(x) = ?$, $\lim\limits_{x\to 3^-} f(x) = ?$

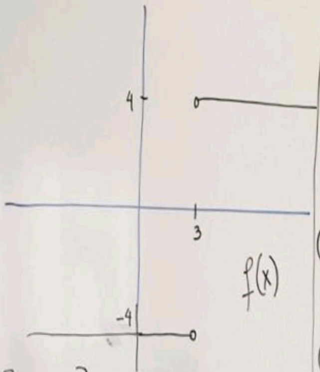
of $f(x) = \dfrac{4x-12}{|x-3|}$ and sketch the graph of $f(x)$.

$|x-3| = \begin{cases} x-3, & x>3 \\ -(x-3), & x<3 \end{cases}$

$\lim\limits_{\substack{x\to 3^+ \\ (x>3)}} f(x) = \lim\limits_{\substack{x\to 3^+ \\ (x>3)}} \dfrac{4x-12}{(x-3)} = \lim\limits_{\substack{x\to 3^+ \\ x\neq 3}} \left[\dfrac{4\cancel{(x-3)}}{\cancel{(x-3)}}\right] = \boxed{4}$

$\lim\limits_{\substack{x\to 3^- \\ (x<3)}} f(x) = \lim\limits_{\substack{x\to 3^+ \\ (x>3)}} \dfrac{4x-12}{-(x-3)} = \lim\limits_{x\to 3^+} \left[\dfrac{4(x-3)}{-(x-3)}\right] = \boxed{-4}$

$\neq \Rightarrow \lim\limits_{x\to 3} f(x) = \text{d.n.e.}$



Ex.5.3: $f(x) = \begin{cases} 2-x^2, & x<0 \\ 7, & x=0 \\ e^x+e^{-x}, & x>0 \end{cases}$

$\lim\limits_{\substack{x\to 0^- \\ (x<0)}} f(x) = \lim\limits_{x\to 0^-} (2-x^2) = \boxed{2}$

$\lim\limits_{\substack{x\to 0^+ \\ (x>0)}} f(x) = \lim\limits_{x\to 0^+} (e^x+e^{-x}) = e^0 + e^{-0} = \boxed{2}$

$\Rightarrow \boxed{\lim\limits_{x\to 0} f(x) = 2}$