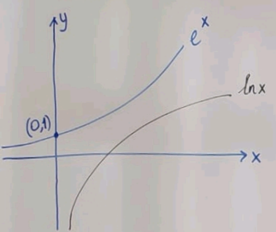
Ex 5.4:



Find the limit $\lim\limits_{\substack{x\to0^+\\(x>0)}} (\ln x) = ?$

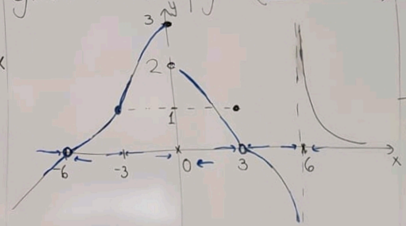
Examining the graph of $y=\ln x$

$\Rightarrow \lim\limits_{x\to0^+}(\ln x) = \boxed{-\infty}$

meaningless

$\lim\limits_{x\to0^-} \ln x$: can not consider this limit, since the domain of $y=\ln x$ is $\boxed{(0,\infty)}$

Ex 5.5: Find the limits based on the function (graph) $f(x)$ given in the figure (if it exists):



a) $\lim\limits_{\substack{x\to-6^-\\(x<-6)}} f(x) = 0$, $\lim\limits_{\substack{x\to-6^+\\(x>-6)}} f(x) = 0 \Rightarrow \lim\limits_{x\to-6} f(x) = 0$ (since $\lim\limits_{x\to-6^-} f(x) = 0 = \lim\limits_{x\to-6^+} f(x)$)

(but $f(-6)$ is not defined).

b) $\lim\limits_{\substack{x\to-3^-\\(x<-3)}} f(x) = 1$, $\lim\limits_{\substack{x\to-3^+\\(x>-3)}} f(x) = 1 \Rightarrow \lim\limits_{x\to-3} f(x) = 1$

c) $\lim\limits_{\substack{x\to0^-\\(x<0)}} f(x) = 3$, $\lim\limits_{\substack{x\to0^+\\(x>0)}} f(x) = 2 \Rightarrow \lim\limits_{x\to0} f(x) =$ d.n.e.

(since left and right limits at $x=0$ are not the same)

d) $\lim\limits_{\substack{x\to3^-\\(x<3)}} f(x) = 0$, $\lim\limits_{\substack{x\to3^+\\(x>3)}} f(x) = 0 \Rightarrow \lim\limits_{x\to3} f(x) = 0$

e.) $\lim\limits_{\substack{x\to 6^-\\(x<6)}} f(x) = -\infty$, $\lim\limits_{\substack{x\to 6^+\\(x>6)}} f(x) = \infty$

$\Downarrow$

$\lim\limits_{x\to 6} f(x) = \text{d.n.e.}$

Ex. 5-6: Evaluate the limit (if it exists):

$$\lim_{x\to 8^+}\left(\frac{x^2-10x+16}{\sqrt{x-8}}\right) = \frac{0}{0}$$

$$x^2-10x+16 = (x-8)(x-2)$$
$$= (\sqrt{x-8})(\sqrt{x-8})(x-2)$$

$$= \lim_{\substack{x\to 8^+\\(x>8)}} \frac{(\cancel{\sqrt{x-8}})(\sqrt{x-8})(x-2)}{\cancel{\sqrt{x-8}}} = (0)(6) = \boxed{0}$$

$x\to 8^+ \Rightarrow x\neq 8$

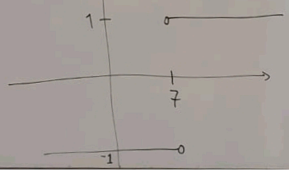
p. 12 Evaluate the following limits (if any):

5.5) $\lim\limits_{x\to 7^-} \frac{|x-7|}{(x-7)} = \frac{0}{0}$ (indeterminate form)

$$|x-7| = \begin{cases} x-7, & x>7 \\ -(x-7), & x<7 \end{cases}$$

$$\lim_{\substack{x\to 7^-\\(x<7)}}\left(\frac{|x-7|}{x-7}\right) = \lim_{\substack{x\to 7^-\\x\neq 7\\(x<7)}}\left[\frac{-\cancel{(x-7)}}{\cancel{x-7}}\right] = \boxed{-1}$$

5.6) $\lim\limits_{\substack{x\to 7^+\\(x>7)}}\left(\frac{|x-7|}{x-7}\right) = \lim\limits_{x\to 7^+} \frac{\cancel{(x-7)}}{\cancel{(x-7)}} = \boxed{1}$



$\Rightarrow \lim\limits_{x\to 7} \frac{|x-7|}{x-7} = \text{d.n.e.}$ (since left & right limits are not the same at $x=7$)

5.11) $\lim\limits_{\substack{x\to 0^+\\(x>0)}}\left(\frac{2x^2+3x|x|}{x|x|}\right) = \lim\limits_{x\to 0^+}\left(\frac{2x^2+3x(x)}{x(x)}\right) = \lim\limits_{\substack{x\to 0^+\\(x\neq 0)}} \frac{5\cancel{x^2}}{\cancel{x^2}} = \boxed{5}$

$x>0 \Rightarrow |x| = x$

5.12) $\lim\limits_{\substack{x\to 0^-\\(x<0)}}\left(\frac{2x^2+3x|x|}{x|x|}\right) = \lim\limits_{x\to 0^-}\left(\frac{2x^2-3x^2}{-x^2}\right) = \lim\limits_{\substack{x\to 0^-\\(x\neq 0)}}\left(\frac{\cancel{-x^2}}{\cancel{-x^2}}\right) = \boxed{1}$

$x<0 \Rightarrow |x| = -x$

5.8) $\lim\limits_{\substack{x\to 0^+\\(x>0)}}\left(\frac{\sqrt{16+3x}-4}{x}\right) = \frac{0}{0}$

$$= \lim_{x\to 0^+}\left[\frac{\sqrt{16+3x}-4}{x}\cdot\frac{(\sqrt{16+3x}+4)}{(\sqrt{16+3x}+4)}\right] = \lim_{\substack{x\to 0^+\\x\neq 0}} \frac{\overbrace{(\cancel{16}+3x)-\cancel{16}}^{3\cancel{x}}}{\cancel{x}(\sqrt{16+3x}+4)} = \lim_{x\to 0^+}\left(\frac{3}{\sqrt{16+3x}+4}\right)$$

$$= \frac{3}{4+4} = \boxed{\frac{3}{8}}$$

Continuity:

We say that f is continuous at $x=a$ if:

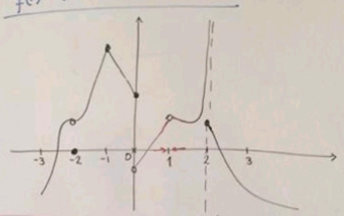
$$\boxed{\lim_{x\to a} f(x) = f(a)}$$

In other words:

- f must be defined at "a"
- limit should exist at "a"
  (i.e. $\lim_{x\to a^-} f(x) = \lim_{x\to a} f(x)$)
- limit at $x=a$ should be equal to the function value at $x=a$.

* A function that is not continuous at $x=a$ is called discontinuous there.

Ex.5.7: Determine the points where $f(x)$ is discontinuous:



$\boxed{x=-2}$: $\lim_{x\to -2} f(x) \neq f(2)$
(limit and function values are different)

$\boxed{x=0}$: limit d.n.e. at $x=0$

$\boxed{x=1}$: f is not defined at $x=1$
($f(1)$ is not defined)

$\boxed{x=2}$: $\lim_{\substack{x\to 2^- \\ (x<2)}} f(x)$ = d.n.e.

Ex.5.8:

Let $f(x) = \begin{cases} 2x^2+a & \text{if } x<2 \\ b & \text{if } x=2 \\ 3x-2 & \text{if } x>2 \end{cases}$

Find "a" and "b" if $f(x)$ is continuous at $x=2$.

$\lim_{\substack{x\to 2^- \\ (x<2)}} f(x) = \lim_{x\to 2^-} (2x^2+a)$
$= 2(2)^2 + a = \boxed{8+a}$

$\lim_{\substack{x\to 2^+ \\ (x>2)}} f(x) = \lim_{x\to 2^+} (3x-2) = 3(2)-2 = \boxed{4}$

In order to have a limit at $x=2$ we must have:

$$\underbrace{\lim_{x\to 2^-} f(x)}_{8+a} = \underbrace{\lim_{x\to 2^+} f(x)}_{4}$$

$$\Rightarrow \boxed{a=-4}$$

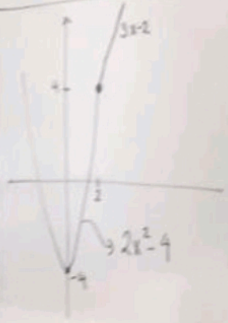
For continuity at $x=2$:

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = \lim_{x\to 2} f(x) = \boxed{4}$$

Moreover; for continuity:

$$b = f(2) = \lim_{x\to 2} f(x) = 4$$

$$\Rightarrow \boxed{b=4}$$



p.43: Find the values of constants that will make the following functions continuous everywhere:

5.21)

$$f(x) = \begin{cases} a+bx^2 & \text{if } x<0 \\ b & \text{if } x=0 \\ 2+e^{-x} & \text{if } x>0 \end{cases}$$

$$\lim_{\substack{x\to 0^- \\ (x<0)}} f(x) = \lim_{x\to 0^-} (a+bx^2) = \boxed{a}$$

$$\lim_{\substack{x\to 0^+ \\ (x>0)}} f(x) = \lim_{x\to 0^+} (2+e^{-x}) = 2+\underbrace{e^{-0}}_{1} = \boxed{3}$$

For continuity at $x=0$ we should have:

$$\left.\underbrace{\lim_{x\to 0^-} f(x)}_{a} = \lim_{x\to 0^+} f(x) = 3\right\} \boxed{a=3}$$

$$\Rightarrow \lim_{x\to 0} f(x) = 3$$

Furthermore; $\underbrace{f(0)}_{b} = \lim_{x\to 0} f(x) = 3$

for cont. at $x=0$

$$\Rightarrow \boxed{b=3}$$

Ans.: $\boxed{a=b=3}$

5.24)

$$f(x) = \begin{cases} e^{ax} & , \text{if } x\leq 0 \\ \ln(b+x^2) & , \text{if } x>0 \end{cases}$$

$$f(0) = e^{a(0)} = \boxed{1}$$

$$\lim_{\substack{x\to 0^- \\ (x<0)}} f(x) = \lim_{\substack{x\to 0^- \\ (x<0)}} (e^{ax}) = e^0 = \boxed{1}$$

$$\lim_{\substack{x\to 0^+ \\ (x>0)}} f(x) = \lim_{x\to 0^+} (\ln(b+x^2)) = \boxed{\ln b} = 1 = \lim_{x\to 0^-} f(x)$$

for cont. at $x=0$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

$$\Rightarrow \ln b = 1 \Leftrightarrow \boxed{b=e}$$

Since for any $a\in\mathbb{R}$.

$$f(0) = e^{a(0)} = 1 = \lim_{x\to 0} f(x)$$

$\Rightarrow \boxed{a}$ is an arbitrary real number

$\log 10 = 1$

$\ln e = 1$

$\log_a a = 1$

$a>0,\ a\neq 1$