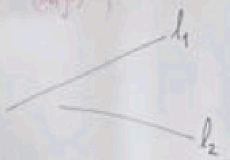
Lines:

$m \to$ slope

$(x_0, y_0) \to$ point



- $l_1 // l_2$: parallel lines
  iff $\boxed{m_{l_1} = m_{l_2}}$ (slopes are equal)
- $l_1 \perp l_2$: perpendicular (orthogonal) lines
  iff $m_1 \cdot m_2 = -1$ (i.e. $m_1 = -\frac{1}{m_2}$)

p.10 / (1-34) $\overbrace{\text{eqn. of the line}}^{l_1}$ that passes through $\boxed{(0,-3)}$ and parallel to the line l_2:

y-intercept

$\boxed{l_1 // l_2 \Rightarrow m_{l_1} = m_{l_2}}$

$\boxed{10y - 5x = 99}$

$\Downarrow$

$y = mx + n$

$\to 10y = 5x + 99$

$\boxed{y = \boxed{\frac{1}{2}}x + \frac{99}{10}}$

$\to m_{l_2}$

l_1:

$m_{l_1} = \frac{1}{2}$

pt. on l_1: $(0,-3)$

$\boxed{y - (-3) = \left(\frac{1}{2}\right)(x - 0)}$

$\boxed{y = \frac{1}{2}x - 3}$

1-35) l_1 : passes through $\boxed{(9,12)}$

$l_1 \perp l_2$: $\underbrace{2x + 5y = 60}$

$5y = -2x + 60$

l_2 : $\boxed{y = \underbrace{\left(-\frac{2}{5}\right)}x + 12}$

$\boxed{m_{l_2} = -\frac{2}{5}}$

$l_1 \perp l_2 \Rightarrow \boxed{m_{l_1}} = -\frac{1}{m_{l_2}} = -\frac{1}{\left(-\frac{2}{5}\right)} = \boxed{\frac{5}{2}}$

$\boxed{l_1 : m_{l_1} = \frac{5}{2},\ \underbrace{P_0(x_0, y_0)}_{(9,12)} \in l_1}$

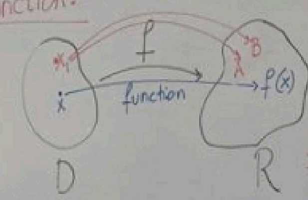
$\Rightarrow y - 12 = \left(\frac{5}{2}\right)(x - 9)$

$2y - 24 = 5x - 45$

$\Rightarrow \boxed{2y - 5x = -21} \Rightarrow \boxed{5x - 2y = 21}$

linear eqn : $\boxed{ax + by = C}$

Function:



$A \neq B$

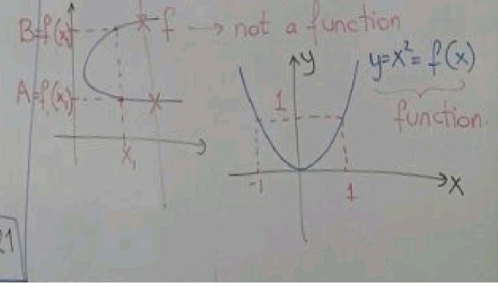
$f(x_1)$

$\Rightarrow$

f not a function

$x_1 \to f_1(x_1) = f_2(x_2)$

not a function



Domain: set of all x values for which $f(x)$ is defined is called the domain of f.

eg i) $f(x)=x^2+5$ is defined for all $x \in \mathbb{R}$ (reals)

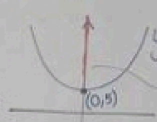
D_f = domain of f $= \boxed{(-\infty, \infty) = \mathbb{R}}$

ii) $g(x) = \dfrac{1}{x+5}$ is defined for all x values except $x = -5$

$\Rightarrow D_g$ = domain of g $= \{x \mid x \neq -5\} = \boxed{(-\infty,-5) \cup (-5,\infty)}$

Range: the set of all $f(x)$ values is called the range of f
(i.e. Range f is a subset of the y-axis)

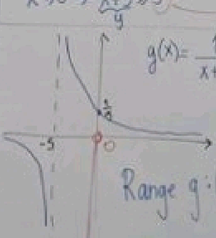
$y = x^2+5$



Range f: $[5,\infty)$

$x^2 \geq 0 \Rightarrow \underbrace{x^2+5}_{y} \geq 5$

$g(x) = \dfrac{1}{x+5}$



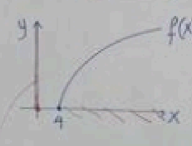
Range g: $(-\infty,0) \cup (0,\infty)$

Ex: $\boxed{f(x)=\sqrt{x-4}}$ Find the domain and range of $f(x)$.

f is defined for all x's for which $x-4 \geq 0 \Rightarrow x \geq 4$

$\Longrightarrow \boxed{\text{Dom } f} = \{x \mid x \geq 4\} = \boxed{[4,\infty)}$

$f(x)=\sqrt{x-4}$



Range $f = [0,\infty)$

$x \geq 4 \Rightarrow x-4 \geq 0 \Rightarrow \underbrace{f(x)=\sqrt{x-4} \geq 0}$

$\Downarrow$

$y \geq 0 \;\; \forall x \geq 4$

$\Downarrow$

$\boxed{\text{Range } f = [0,\infty)}$

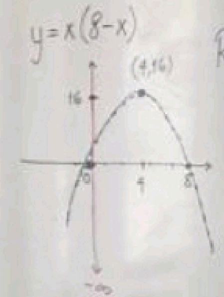
Find the domain and range of the following functions:

1-38) $f(x)=8x-x^2 \Rightarrow$ defined for all x's

$\Rightarrow Dom_f : (-\infty,\infty)=\mathbb{R}$

$y=x(8-x)$

Range: $(-\infty,16]$ y-values



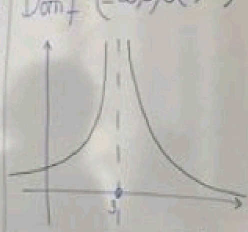
$y=-x^2+8x=-[(x-4)^2-16]=\boxed{-(x-4)^2+16}$

$=-(x^2-8x)$

1-39) $f(x)=\dfrac{1}{x^2-6x+9}$

$=\dfrac{1}{(x-3)^2}$

Dom f: $(-\infty,3)\cup(3,\infty)$



$\boxed{\text{Range } f: (0,\infty)}$

$(x-3)^2\geq 0 \Rightarrow \dfrac{1}{(x-3)^2}>0$

Parabolas:

Quadratic eqn: $\boxed{ax^2+bx+c=0}$ (*)

Quadratic formula: $x_{1,2}=\dfrac{-b\mp\sqrt{b^2-4ac}}{2a}$ (**)

$\Delta=b^2-4ac$, $(a\neq 0)$

↳ discriminant

- if $\Delta>0 \Rightarrow$ (*) has two distinct real roots x_1 & x_2 given by (**)
- if $\Delta=0 \Rightarrow$ (*) has 2 identical real roots $x_1=x_2=-\dfrac{b}{2a}$
- if $\Delta<0 \Rightarrow$ (*) has no real roots

$\Rightarrow$ (*) has no real solution.

Ex 2.2: Solve $8x^2-6x-5=0$

$a=8$, $b=-6$, $c=-5$

$x_{1,2}=\dfrac{-(-6)\mp\sqrt{(-6)^2-4(8)(-5)}}{2(8)}$

$\Delta=(-6)^2-4(8)(-5)$
$=36+160$
$=196>0$

$=\dfrac{6\mp14}{16}=\begin{cases}\dfrac{20}{16}=\boxed{\dfrac{5}{4}}\\[4pt] -\dfrac{8}{16}=\boxed{-\dfrac{1}{2}}\end{cases}$

Soln. of the given quadratic eqn is:

$\left\{\dfrac{5}{4},-\dfrac{1}{2}\right\}$ or $x_1=\dfrac{5}{4}$, $x_2=-\dfrac{1}{2}$

$(x-x_1)(x-x_2)=0$ $(\Rightarrow 8x^2-6x-5=0)$

$(4x-5)(2x+1)=0$

$8x^2-6x-5=0$ ✓

Ex 2.3: Solve $9x^2-12x+4=0$

$a=9, b=-12, c=4$

$\Delta = b^2-4ac = (-12)^2-4(9)(4)$

$= 144-144=0$

$\Rightarrow$ 2 repeated (identical) real roots

$\Rightarrow x_1 = x_2 = \frac{-b}{2a} = \frac{-(-12)}{2(9)} = \frac{12}{18} = \frac{2}{3}$

Soln: $\{\frac{2}{3}\}$

$(3x-2)^2 = 9x^2-12x+4=0$

✓

Ex 2.4: Solve $3x^2+6x+4=0$

$\Delta = b^2-4ac = (6)^2-4(3)(4)$

$=36-48<0$

$\Rightarrow$ no real solution of the given quadratic eqn. exists!

$\Rightarrow$ Solution: $\{\phi\}$