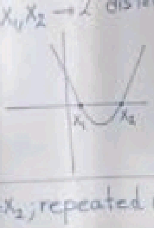
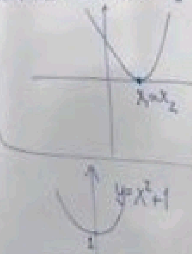
Quadratic eqn.: $\boxed{ax^2+bx+c=0}$ (*)

$\Delta = b^2 - 4ac$ $(a>0)$

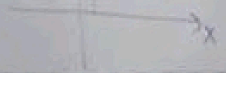
* $\Delta > 0$: $x_1, x_2 \to 2$ distinct real roots of (*)

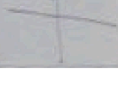


* $\Delta = 0$: $x_1 = x_2$; repeated real root of (*)



* $\Delta < 0 \Rightarrow$ no real root (soln.) of *





Quadratic function:

A function of the form: $\boxed{\underbrace{ax^2+bx+c}_{f(x)}}$, $a \neq 0$

is called a quadratic function.

The graph of a quadratic function is a parabola; and

* if $a>0 \Rightarrow$ parabola opens upward $(\cup)$

* if $a<0 \Rightarrow$ parabola opens downward $(\cap)$

$\Rightarrow$ The vertex of the parabola is either the maximum or the minimum pt. on the graph of the parabola

The x-coordinate of the vertex is $-\frac{b}{2a}$, and the y-coordinate is $f\left(-\frac{b}{2a}\right) \Rightarrow \boxed{\text{Vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)}$

$$\boxed{f(x) = ax^2+bx+c} = a\left[\underbrace{x^2+\frac{b}{a}x}_{\left(x+\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2}+\frac{c}{a}\right]$$

$$= a\left[\left(x+\frac{b}{2a}\right)^2+\left(\frac{c}{a}-\frac{b^2}{4a^2}\right)\right]$$

$$f(x) = \boxed{a\left(x+\frac{b}{2a}\right)^2+\left(c-\frac{b^2}{4a}\right)}$$

Ex 2-5: Sketch the graph of $\boxed{f(x) = x^2 - 10x + 16}$

$a=1$, $b=-10$, $c=16$

$f(5) = 25-50+16 = \boxed{-9}$

$a = 1 > 0 \Rightarrow (\cup)$ ✓

Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \left(-\frac{-10}{2(1)}, f(5)\right) = \boxed{(5,-9)}$ ✓

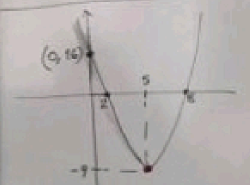
($-\frac{-10}{2(1)} = 5$)

Alternatively, by completing to square the quadratic function:

we get:

$\boxed{f(x)} = (x-5)^2 - 25 + 16 = \boxed{(x-5)^2 - 9}$

$\Rightarrow$ Vertex: $(5,-9)$



$f(0)=16 \rightarrow$ $\boxed{\text{y-intercept: } (0,16)}$

$x^2 - 10x + 16 = 0$ ✓

$(x-2)(x-8) = 0$

$x=2, x=8$

$\boxed{\text{x-intercept(s): } (2,0), (8,0)}$ ✓

Find the vertex, and x- and y-intercepts of the following parabolas. Sketch their graph.

2-12) $y = -x^2 + 12$

$a = -1 < 0 \Rightarrow \cap$

$b = 0, c = 12$

$\boxed{\text{Vertex}}$ $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \boxed{(0,12)}$

intercepts:

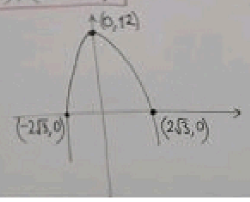
y-intercept: $x = 0 \Rightarrow y = 12 \Rightarrow \boxed{(0,12)}$

x-intercepts (if any): $-x^2 + 12 = 0$

$x^2 = 12 \Rightarrow x = \mp\sqrt{12}$

$\boxed{x = \mp 2\sqrt{3}}$

$\boxed{(2\sqrt{3},0), (-2\sqrt{3},0)}$



2-15) $\boxed{y = x^2 + 10x + 25} = f(x)$

$a = 1 > 0 \Rightarrow \cup$

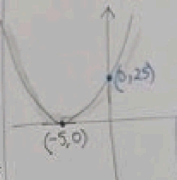
$a=1, b=10, c=25 \Rightarrow -\frac{b}{2a} = -\frac{10}{2} = \boxed{-5}$

$y = \underbrace{(x+5)^2}_{=0} \Rightarrow \underbrace{x_0 = -5 \text{ double root}}$

$\boxed{(-5,0): \text{ x-intercept}}$

y-intercept: $f(0) = 25$

$\boxed{(0,25)}$



$\boxed{\text{Vertex: } (-5,0)}$

Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = (-5,0)$

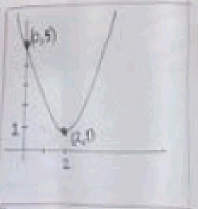
$x^2 + 2x + 1 = (x+1)^2$

2-19) $y = x^2 - 4x + 5$

$a = 1 > 0 \Rightarrow \cup$

$b = -4,\ c = 5$

$y = (x-2)^2 - 4 + 5 = \boxed{(x-2)^2 + 1}$



$\boxed{\text{Vertex: } (2,1)}$

Intercepts:

y-intercept: $x = 0 \Rightarrow y = 5$

$\boxed{(0,5)}$

x-intercept(s): $x^2 - 4x + 5 = 0$

$(x-2)^2 + 1 = 0$

$(x-2)^2 = -1 \rightarrow$ impossible

no solution

$\boxed{\text{no x-intercepts}}$

$\Downarrow$

graph does not intersect the x-axis.

2-20) $\boxed{y = -3x^2 + 60x - 450} = -3[x^2 - 20x + 150]$

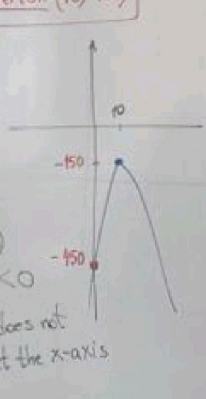
$a = -3 < 0 \Rightarrow \cap$

$b = 60 \Rightarrow -\frac{b}{2a} = -\frac{60}{2(-3)} = 10$

$f(10) = -3(100) + 60(10) - 450$

$= -750 + 600 = \boxed{-150}$

$\boxed{\text{Vertex: } (10, -150)}$



y-intercept: $(0, -450)$

x-intercepts:

$-3x^2 + 60x - 450 = 0$

$\Delta = b^2 - 4ac = (60)^2 - 4(-3)(-450)$

$= 3600 - 5400 < 0$

$\Rightarrow$ no real roots $\Rightarrow$ graph does not intersect the x-axis

$\Rightarrow \boxed{\text{no x-intercepts}}$

Polynomials (p.21):

A function of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 \qquad n \in \mathbb{Z}_+$$

is called a polynomial of degree n.

For example; $10x^5 - 7x + \frac{7}{2}$ is a polynomial of degree 5.

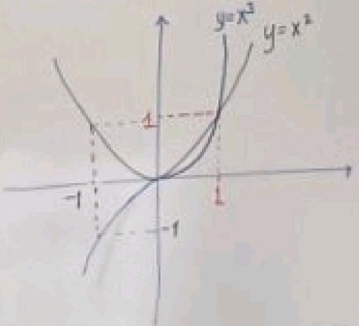
But; $\sqrt{x}, x^{-1}, \frac{1}{1+x}, x^{5/3}$ are not polynomials.

Rational functions:

The quotient of two polynomials is a rational function

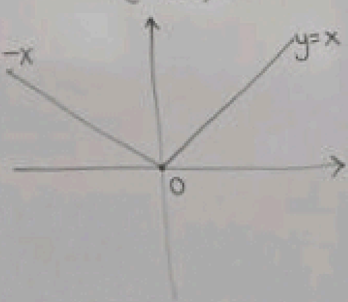
$$\boxed{f(x) = \frac{p(x)}{q(x)}} \rightarrow \text{polynomials}$$

eg. $\boxed{\frac{3x^2 - 5}{1 + 2x - 7x^3}} \rightarrow$ rational function

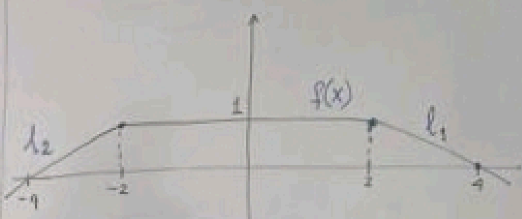


Piecewise-defined functions:

$$|x|=\begin{cases} x, & x\geqslant 0 \\ -x, & x<0 \end{cases}$$



Ex 3-2: Find the formula of the function $f(x)$:



$$f(x)=\begin{cases} \frac{1}{2}x+2 & x<-2 \\ 1 & -2\leqslant x\leqslant 2 \\ -\frac{1}{2}x+2 & x>2 \end{cases}$$

$x>2$:

$m_1=\frac{1-0}{2-4}=-\frac{1}{2}$

$y-0=(-\frac{1}{2})(x-4)$

$\boxed{y=-\frac{1}{2}x+2}$

$x<-2$:

$m_2=\frac{1-0}{-2-(-4)}=\boxed{\frac{1}{2}}$

l_2: $y-0=(\frac{1}{2})(x-(-4))$

$\boxed{y=\frac{1}{2}x+2}$

p. 25/3.2 Sketch the graph:

$$f(x)=\begin{cases} x+3, & \text{if } x<4 \\ x-1, & \text{if } x\geqslant 4 \end{cases}$$