p.33/4.18 $$\lim_{x\to\infty}\left\{\underbrace{\frac{1}{2x-\sqrt{4x^2-5x+6}}}_{\frac{1}{\infty-\infty}\ \text{indeterminate form}}\right\}=?$$

$$=\lim_{x\to\infty}\left[\frac{2x+\sqrt{4x^2-5x+6}}{\left(2x-\sqrt{4x^2-5x+6}\right)\left(2x+\sqrt{4x^2-5x+6}\right)}\right]$$

$$=\lim_{x\to\infty}\frac{\left(2x+\overbrace{\sqrt{4x^2-5x+6}}^{\sqrt{x^2\left(4-\frac{5}{x}+\frac{6}{x^2}\right)}}\right)}{\underbrace{4x^2-\left(4x^2-5x+6\right)}_{5x-6}}$$

$$=\lim_{x\to\infty}\left[\frac{2x+x\sqrt{4-\frac{5}{x}+\frac{6}{x^2}}}{5x-6}\right]$$

$$=\lim_{x\to\infty}\frac{x\left[2+\sqrt{4-\underbrace{\frac{5}{x}}_{0}+\underbrace{\frac{6}{x^2}}_{0}}\right]}{x\left[5-\underbrace{\frac{6}{x}}_{0}\right]}$$

$$=\frac{2+\sqrt{4-0+0}}{5-0}=\frac{2+2}{5}=\boxed{\frac{4}{5}}$$

4.16) $$\lim_{x\to\infty}\left[\frac{(x^4-16)}{(2x-1)(2x+1)(x^2+1)}\right]=?$$

$$=\lim_{x\to\infty}\left[\frac{(x^4-16)}{\underbrace{(4x^2-1)(x^2+1)}_{4x^4+3x^2-1}}\right]$$

$$=\lim_{x\to\infty}\left(\frac{x^4-16}{4x^4+3x^2-1}\right)$$

$$=\lim_{x\to\infty}\frac{x^4\left[1-\boxed{\frac{16}{x^4}}\right]}{x^4\left[4+\underbrace{\boxed{\frac{3}{x^2}}-\boxed{\frac{1}{x^4}}}_{0\quad 0}\right]}=\frac{1-0}{4+0+0}=\boxed{\frac{1}{4}}$$

because of (A)

4.13) $$\lim_{x\to\infty}\frac{2+3x-4x^4}{\sqrt{x}\left(1-17x+8x^3\right)}=?$$

$$=\lim_{x\to\infty}\frac{\left(2+3x-4x^4\right)}{x^{1/2}-17x^{3/2}+8x^{7/2}}=\lim_{x\to\infty}\frac{x^4\left(\frac{2}{x^4}+\frac{3}{x^3}-4\right)}{x^{7/2}\left(\frac{1}{x^3}-\frac{17}{x^2}+8\right)}$$

$\sqrt{x}$

$$=\underbrace{\left(\lim_{x\to\infty}\sqrt{x}\right)}_{\infty}\underbrace{\left(\frac{0+0-4}{0-0+8}\right)}_{\left(-\frac{1}{2}\right)}=\boxed{-\infty}$$

4.12) $\lim_{x\to\infty}\left[\frac{3x^2+12x+9}{(x^2-1)(x^2+1)}\right]=?$

$=\lim_{x\to\infty}\frac{(3x^2+12x+9)}{(x^4-1)}$

$=\lim_{x\to\infty}\frac{\cancel{x^2}\left(3+\frac{12}{x}+\frac{9}{x^2}\right)}{\underbrace{\cancel{x^4}}_{x^2}\left(1-\frac{1}{x^4}\right)}$

$=\left(\lim_{x\to\infty}\left(\frac{1}{x^2}\right)\right)\left(\lim_{x\to\infty}\left(\frac{3+\overbrace{\frac{12}{x}}^{0}+\overbrace{\frac{9}{x^2}}^{0}}{1-\underbrace{\frac{1}{x^4}}_{0}}\right)\right)$

$=\underbrace{\left(\frac{1}{\infty}\right)}_{0}\underbrace{\left(\frac{3+0+0}{1-0}\right)}_{3}=(0)(3)=\boxed{0}$

4.38) $\lim_{x\to\infty}\left(\frac{1}{\ln(x^2)}\right)=\frac{1}{\ln(\infty)}=\frac{1}{\infty}=\boxed{0}$

4.39) $\lim_{x\to\infty}\left(\frac{8e^x}{4+5e^x}\right)=?$

$=\lim_{x\to\infty}\left[\frac{\cancel{e^x}(8)}{\cancel{e^x}\left(\underbrace{\frac{4}{e^x}}_{0}+5\right)}\right]=\frac{8}{0+5}=\boxed{\frac{8}{5}}$

$\left(\frac{4}{\infty}=0\right)$

4.40) $\lim_{x\to\infty}\left(\sqrt{2x^2-1}-\sqrt{x^2+1}\right)\frac{\left(\sqrt{2x^2-1}+\sqrt{x^2+1}\right)}{\left(\sqrt{2x^2-1}+\sqrt{x^2+1}\right)}=\lim_{x\to\infty}\frac{\overbrace{(2x^2-1)-(x^2+1)}^{(x^2-2)}}{x\left(\sqrt{2-\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)}$

$\left(x\sqrt{2-\frac{1}{x^2}}+x\sqrt{1+\frac{1}{x^2}}\right)$

$=\lim_{x\to\infty}\frac{\overset{x}{\cancel{x^2}}\left(1-\frac{2}{x^2}\right)}{\cancel{x}\left(\sqrt{2-\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)}=\left(\lim_{x\to\infty}x\right)\left(\frac{1-0}{\sqrt{2-0}+\sqrt{1+0}}\right)=(\infty)\left(\frac{1}{\sqrt{2}+1}\right)=\boxed{+\infty}$

$\lim_{x\to+\infty}\sqrt{x^2}=\lim_{x\to+\infty}(x)$

$\lim_{x\to-\infty}\sqrt{x^2}=\lim_{x\to-\infty}(-x)$

$\sqrt{x^2}=|x|=\begin{cases}x, & x\geq 0\\ -x, & x<0\end{cases}$

A

$\boxed{\lim_{x\to\mp\infty}\left(\frac{c}{x^p}\right)=0}$

c→constant, $p>0$

$\frac{0}{0},\frac{\infty}{\infty},\infty-\infty$

$\underbrace{\frac{1}{\infty-\infty}}$

indeterminate form

p(x), q(x) → polynomials

$\lim_{x\to\infty}\left(\frac{p(x)}{q(x)}\right)=\begin{cases}\infty & \text{if } \deg p(x)>\deg q(x)\\ * & \text{if } \boxed{\deg p(x)=\deg q(x)}\\ 0 & \text{if } \deg p(x)<\deg q(x)\end{cases}$

* $\frac{\text{leading term coeff of } p(x)}{\text{" " " " } q(x)}$

4.31) $\lim\limits_{x\to 0}\left[\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}\right]=\dfrac{0}{0}$ (indeterminate form)

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x}-\sqrt{1-x}\right)}{x}\cdot\frac{\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$=\lim_{x\to 0}\frac{\overbrace{(1+x)-(1-x)}^{2x}}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$=\lim_{\substack{\boxed{x\to 0}\\(x\neq 0)}}\left(\frac{2\cancel{x}}{\cancel{x}\left(\sqrt{1+x}+\sqrt{1-x}\right)}\right)$$

$$=\frac{2}{1+1}=\boxed{1}$$

$\boxed{x^4-c^4}=(x^2-c^2)(x^2+c^2)$
$=(x-c)(x+c)(x^2+c^2)$

$x^3-c^3=(x-c)(x^2+xc+c^2)$

4.35) $\lim\limits_{x\to c}\left(\dfrac{x^4-c^4}{x^3-c^3}\right)=\lim\limits_{\substack{x\to c\\x\neq c}}\dfrac{\cancel{(x-c)}(x+c)(x^2+c^2)}{\cancel{(x-c)}(x^2+xc+c^2)}$

$$=\lim_{x\to c}\left[\frac{(x+c)(x^2+c^2)}{(x^2+xc+c^2)}\right]=\frac{\overbrace{(c+c)}^{2c}\overbrace{(c^2+c^2)}^{2c^2}}{\underbrace{c^2+c\cdot c+c^2}_{3c^2}}$$

$$=\frac{4c^3}{3c^2}=\boxed{\frac{4}{3}c}$$

$x^2-1=(x-1)(x+1)$
$x^3-1=(x-1)(x^2+x+1)$
$x^4-1=(x-1)(x+1)(x^2+1)$
$=(x-1)(x^3+x^2+x+1)$
$x^5-1=(x-1)(x^4+x^3+x^2+x+1)$

4.36) $\lim\limits_{x\to 0}\left(\dfrac{x}{\sqrt{a+bx}-\sqrt{a-cx}}\right)=\dfrac{0}{\sqrt{a+0}-\sqrt{a-0}}=\dfrac{0}{0}$ (indeterminate form)

$$=\lim_{x\to 0}\left(\frac{x}{\sqrt{a+bx}-\sqrt{a-cx}}\right)\cdot\left(\frac{\sqrt{a+bx}+\sqrt{a-cx}}{\sqrt{a+bx}+\sqrt{a-cx}}\right)$$

$$=\lim_{x\to 0}\frac{x\left(\sqrt{a+bx}+\sqrt{a-cx}\right)}{\underbrace{\underbrace{(a+bx)-(a-cx)}_{\cancel{a}+bx-\cancel{a}+cx}}_{x(b+c)}}=\lim_{x\to 0}\frac{\cancel{x}\left(\sqrt{a+bx}+\sqrt{a-cx}\right)}{\cancel{x}(b+c)}$$

$$=\frac{\sqrt{a+0}+\sqrt{a-0}}{b+c}=\frac{2\sqrt{a}}{b+c}$$

4-37) $\lim_{x\to 1}\left(\frac{x^n-1}{x-1}\right)=\frac{0}{0}=?\quad (n\in\mathbb{Z}_+)$

$$=\lim_{x\to 1}\frac{\cancel{(x-1)}(x^{n-1}+x^{n-2}+x^{n-3}+\cdots+x^3+x^2+x+1)}{\cancel{(x-1)}}$$

$$=\underbrace{1+1+\cdots+1}_{\text{n-terms}}=\boxed{n}$$

4-34) $\lim_{x\to 1}\left(\frac{1-\sqrt{x}}{1-\sqrt[3]{x}}\right)=\frac{0}{0}=$ indeterminate form

$$\boxed{x=u^6}\Rightarrow \left.\begin{aligned}\sqrt{x}&=u^3\\ \sqrt[3]{x}&=u^2\end{aligned}\right\}\ x\to 1\Rightarrow u\to 1$$

substitution

$$\lim_{u\to 1}\left(\frac{1-u^3}{1-u^2}\right)=\lim_{u\to 1}\frac{\cancel{(1-u)}(1+u+u^2)}{\cancel{(1-u)}(1+u)}=\boxed{\frac{3}{2}}$$

4-32) $\lim_{x\to -2}\left[\frac{(x+2)^2}{x^4-16}\right]=\left(\frac{0}{0}\right)$

$$=\lim_{x\to -2}\left[\frac{\cancel{(x+2)}(x+2)}{\underbrace{(x^2-4)}_{\cancel{(x+2)}(x-2)}(x^2+4)}\right]$$

$$=\lim_{x\to -2}\left[\frac{(x+2)}{(x-2)(x^2+4)}\right]$$

$$=\frac{0}{(-4)((-2)^2+4)}$$

$$=\frac{0}{-32}=\boxed{0}$$

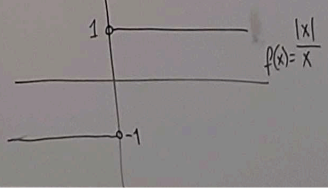
4-23) $\lim_{x\to 0}\frac{|x|}{x}=?$

$x>0\Rightarrow |x|=x\Rightarrow \lim_{\substack{x\to 0\\(x>0)}}\left(\frac{\cancel{x}}{\cancel{x}}\right)=\lim_{x\to 0}(1)=\boxed{1}$

$x<0\Rightarrow |x|=-x\Rightarrow \lim_{\substack{x\to 0\\(x<0)}}\left(\frac{-x}{x}\right)=\lim_{x\to 0}(-1)=\boxed{-1}$

$$\lim_{x\to 0}\frac{|x|}{x}=\text{d.n.e.}$$

(since limits as $x>0$ and $x<0$ are not the same)



4-28) $\lim_{x\to 2}\frac{x^2-7x+10}{x^2-5x+6}\quad \neq\frac{1}{1}=1$

$$=\lim_{x\to 2}\frac{\cancel{(x-2)}(x-5)}{\cancel{(x-2)}(x-3)}=\frac{2-5}{2-3}=\frac{-3}{-1}=\boxed{3}$$

4-27) $\lim_{x\to 0}\left(\frac{x^4-5x^2+12x+7}{5x^2+6}\right)\ \left(\neq\frac{(\infty)}{(0)}\right)$

~~deg p(x) > deg q(x)~~

$$=\boxed{\frac{7}{6}}$$