Functions:

Equations: solve for the unknown variable

$8x-2=5x+7 \longrightarrow$ linear (1^{st} degree) eqns.

$8x-5x=7+2$

$\frac{3x}{3}=\frac{9}{3} \Rightarrow \boxed{x=3}$

Ex 1.1: Solve the eqn $\frac{2x}{2x+5}=\frac{3}{4}$

rational function

$4(2x)=3(2x+5)$

$8x=6x+15$

$2x=15 \longrightarrow \boxed{x=\frac{15}{2}}$

Absolute value: $|x|=a \Leftrightarrow x=a$ or $x=-a$

Ex 1.2:

$|3x-12|=27$

$\Rightarrow 3x-12=27$ or $3x-12=-27$

$\Downarrow \qquad\qquad \Downarrow$

$3x=39$ or $3x=-27+12$

$3x=-15$

$\Downarrow$

$\boxed{x=13}$ or $\boxed{x=-5}$

$\boxed{\text{Soln: } \{-5,13\}}$

Intervals:

Closed interval: $[a,b]=\{x \mid a\leq x\leq b\}$

Open interval: $(a,b)=\{x \mid a<x<b\}$

Half-open interval: $[a,b)=\{x \mid a\leq x<b\}$

$(a,b]=\{x \mid a<x\leq b\}$

Unbounded interval:

$(a,\infty)=\{x \mid x>a\}$ (or $[a,\infty)=\{x \mid x\geq a\}$)

$(-\infty,a)=\{x \mid x<a\}$ (or $(-\infty,a]=\{x \mid x\leq a\}$)

$\mathbb{R}$: real line corresponds to the interval $(-\infty,\infty)$.

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Inequalities:

Ex. 1.3: Solve the inequality $7x-5 \leq 30$:

$\left.\frac{7x}{7} \leq 30+5 = \frac{35}{7}\right\} \Rightarrow \boxed{x \leq 5}$

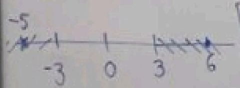
Soln.: $\{x \mid x \leq 5\} \Rightarrow$ 

$(-\infty, 5]$

Ex. 1.4: $\boxed{|x+10| < 11}$

* $|x| \leq a \iff -a \leq x \leq a$

* $|x| > 3 \iff \boxed{x < -3 \text{ or } x > 3}$



$|x| \geq a \iff$
$x \leq -a$ or $x \geq a$

$|x+10| < 11 \Rightarrow$

$-11 < x+10 < 11$

$-11-10 < x < 11-10$

$\boxed{-21 < x < 1} \underset{\text{interval}}{\Longrightarrow} \boxed{(-21, 1)}$

Ex. 1.5: $|x+10| > 11 \Rightarrow$

i) $x+10 > 11$

ii) $-(x+10) > 11 \Rightarrow x+10 < -11$

i) $\Rightarrow x > 11-10 \Rightarrow \boxed{x > 1}$

or

ii) $x < -11-10 \Rightarrow \boxed{x < -21}$

Soln.: $\{x \mid x < -21 \text{ or } x > 1\}$

$\boxed{\text{Soln.: } (-\infty, -21) \cup (1, \infty)}$

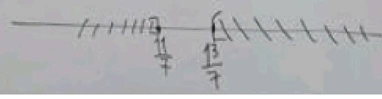
p. 10/1-28) Solve the inequality: $|12-7x| \geq 1$

$\iff 12-7x \geq 1$ or $-(12-7x) \geq 1$ (or $12-7x \leq -1$)

$12-1 \geq 7x$ or $12+1 \leq 7x$

$\frac{11}{7} \geq \frac{7x}{7}$ $\qquad$ $13 \leq 7x$

$\Rightarrow \boxed{x \leq \frac{11}{7}}$ $\qquad$ $\boxed{x \geq \frac{13}{7}}$



Soln.:
$\left(-\infty, \frac{11}{7}\right] \cup \left[\frac{13}{7}, \infty\right)$

Exercises on Algebraic Operations:

1-2) $\left(\frac{1}{16}\right)^{3/4} = \frac{1^{3/4}}{(2^4)^{3/4}} = \frac{1}{2^3} = \boxed{\frac{1}{8}}$

$\boxed{(a)^{3/4} = \sqrt[4]{a^3}}$

$(a)^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$

m,n → integer

1-5) $\sqrt[3]{\frac{8}{1000}} = \left(\frac{2^3}{10^3}\right)^{1/3} = \frac{2^{3/3}}{10^{3/3}} = \frac{2}{10}$

$a^3 + 1^3$

$a^3 + b^3 \quad | \; (a+b)$

$-a^3 + a^2b \qquad \boxed{a^2 - ab + b^2}$

$-a^2b + b^3$

$\mp a^2b \mp ab^2$

$ab^2 + b^3$

$-ab^2 + b^3$

$0 \quad 0$

1-1) $(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 2 & & 1 & \\ & & 1 & & 3 & & 3 & & 1 \\ 1 & & 4 & & 6 & & 4 & & 1 \\ & & & & \vdots & & & & \end{array}$$

Pascal's triangle

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$[a + (-b)]^3 =$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

1-9) $a^2 - b^2 = (a-b)(a+b)$

$\frac{1}{\sqrt{5} - \sqrt{3}} \cdot \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})} = \frac{\sqrt{5}+\sqrt{3}}{5-3} = \boxed{\frac{\sqrt{5}+\sqrt{3}}{2}}$

$\left(= \frac{\sqrt{5}}{2} + \frac{\sqrt{3}}{2}\right)$

1-10) $\frac{12}{\sqrt{7}-1} - \frac{12}{\sqrt{7}+1} = ?$

$(\sqrt{7}+1) \quad (\sqrt{7}-1)$

$= \frac{12(\sqrt{7}+1)}{7-1} - \frac{12(\sqrt{7}-1)}{7-1}$

$= \frac{12\sqrt{7} + 12 - 12\sqrt{7} + 12}{6}$

$= \frac{24}{6} = \boxed{4}$

1-12) $(\sqrt[n]{a^m})(\sqrt[n]{b^k}) = \sqrt[n]{a^m b^k} = a^{m/n} b^{k/n}$

$(\sqrt{x^3 y})(\sqrt{64 x y^9}) = \sqrt{64 x^4 y^{10}}$

$64 = 8^2$

$= 8^{2/2} \cdot x^{4/2} \cdot y^{10/2} = \boxed{8 \cdot x^2 \cdot y^5}$

1-13) $x^3 - 1 = (x-1)(x^2 + x + 1)$

1-14) $(\sqrt{x^2+4} + 3)(\sqrt{x^2+4} - 3) =$

$= (\sqrt{x^2+4})^2 - (3)^2 = x^2 + 4 - 9$

$= \boxed{x^2 - 5}$

1-15) $x^4-100y^4=(x^2)^2-(10y^2)^2=(x^2-10y^2)(x^2+10y^2)$

1-17) $(3a-2b)^2=(3a)^2-2(3a)(2b)+(2b)^2$

$=\boxed{9a^2-12ab+4b^2}$

1-19) $\frac{2x}{x^2-4}+\frac{5}{x+2}=?$

$(x-2)(x+2)$ $\quad(x-2)$

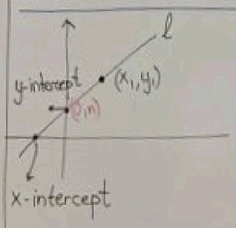
$=\frac{2x}{x^2-4}+\frac{5(x-2)}{(x+2)(x-2)}=\frac{2x+5x-10}{x^2-4}$

$=\boxed{\frac{7x-10}{x^2-4}}$

1-20) $1-\frac{1}{1+\frac{1}{x}}=1-\frac{1}{\frac{x+1}{x}}=1-\frac{x}{x+1}=\frac{x+1-\cancel{x}}{x+1}=\boxed{\frac{1}{x+1}}$

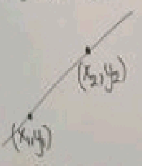
$\left(\frac{a}{\frac{b}{c}}=a\cdot\frac{c}{b}\right)$

Lines on the Plane:



$\text{slope}=m=\frac{\text{change in } y}{\text{change in } x}=\frac{\Delta y}{\Delta x}$

$(x_1,y_1),(x_2,y_2)$ on ℓ



$m=\frac{\Delta y}{\Delta x}=\frac{y_1-y_2}{x_1-x_2}=\frac{y_2-y_1}{x_2-x_1}$

$x=0\to$ gives the y-intercept of the line ℓ

$y=0\to$ " " x-intercept of " "

* slope-intercept eqn: $\boxed{y=mx+n}$ $\quad(0,n)\to$ y-intercept of ℓ
* point-slope eqn: $\boxed{y-y_1=m(x-x_1)}$

1-31) Find the eqn of the line passing through the origin $((x_1,y_1)=(0,0))$ and has slope $m=\frac{1}{5}$

$\Rightarrow y-0=\frac{1}{5}(x-0)\Rightarrow\boxed{y=\frac{1}{5}x}$

$\Rightarrow 5y=x\Rightarrow\boxed{x-5y=0}$

linear eqn

$\boxed{ax+by=c}$