Ch 8: Implicit Differentiation:

* $x^2+y^2=1$
* $ye^y+2x-\ln y=0$
* $xe^x+e^y=\sqrt{x+2y}$

$\Big\}$ $y=y(x)$ implicitly

The derivative of y w.r.t. x can be found implicitly following the below guidelines:

* differentiate everything w.r.t. "x"
* solve for y'

steps for implicit differentiation

Ex 8-1: Find y' using the equation

$$\boxed{y+y^3=3x^2+1}$$

$y=y(x)$

$$\boxed{\frac{d}{dx}(y^n)=ny^{n-1}\cdot y'}$$

$$\frac{d}{dx}(y)+\frac{d}{dx}(y^3)=\frac{d}{dx}(3x^2+1)$$

$$\boxed{y'+(3y^2)(y')=6x}$$

$$y'[1+3y^2]=6x$$

$$\boxed{y'=\frac{6x}{1+3y^2}}$$

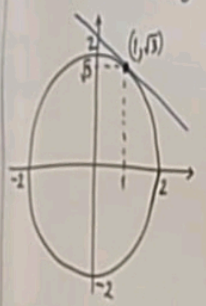
HW: Redo the following examples: 8.3, 8.5, 8.8

Ex 8.2: Find the slope of the tangent line

implicit eqn.

$$y'\Big|_{(x_0,y_0)}=?$$

to the curve: $x^2+y^2=4$ at $(1,\sqrt{3})$, with $x_0 = 1$, $y_0 = \sqrt{3}$.



Alternative method:

$y=\sqrt{4-x^2}$ (explicit eqn.)

$$\Rightarrow \frac{dy}{dx}\Big|_{x=1}=?$$

$$x^2+y^2=4 \Rightarrow 2x+2y\cdot y'=0$$

$$y'=-\frac{2x}{2y}=-\frac{x}{y}\Rightarrow y'\Big|_{(1,\sqrt{3})}=-\frac{1}{\sqrt{3}}=m=$$ slope of tg. line at $(1,\sqrt{3})$

tg. line eqn:

$$(y-\sqrt{3})=\left(-\frac{1}{\sqrt{3}}\right)(x-1)\Rightarrow \boxed{y=-\frac{1}{\sqrt{3}}x+\frac{4}{\sqrt{3}}}$$

Ex 8.4: Find y' (using implicit differentiation); where

$$\boxed{\underbrace{x^2e^y}_{\text{product}}+y=e^{3x}}$$

$y=y(x)$

$$[(2x)(e^y)+(x^2)\cdot(e^y\cdot y')]+y'=e^{3x}\cdot(3)$$

$$\frac{d}{dx}(e^{f(x)})=e^{f(x)}\cdot f'(x)$$

$$y'[x^2e^y+1]=3e^{3x}-2xe^y$$

$$\Rightarrow \boxed{y'=\frac{3e^{3x}-2xe^y}{x^2e^y+1}}$$

Ex 8.6: Find y' where

$$\boxed{\underbrace{ye^{xy}}_{\text{product}} + \underbrace{x^4 \ln x}_{\text{product}} = e^{3x}}$$

$$\left(y' e^{xy} + y\left[e^{xy}\underbrace{\left(\frac{d}{dx}(xy)\right)}_{(1\cdot y + x\cdot y')}\right]\right) + \left[4x^3 \ln x + x^4 \cdot \frac{1}{x}\right] = 3e^{3x}$$

$$y' e^{xy} + ye^{xy}(y + xy') + (4x^3 \ln x + x^3) = 3e^{3x}$$

$$y'\left[e^{xy} + xye^{xy}\right] = 3e^{3x} - y^2 e^{xy} - 4x^3 \ln x - x^3$$

$$\boxed{y' = \frac{3e^{3x} - y^2 e^{xy} - 4x^3 \ln x - x^3}{e^{xy} + xye^{xy}}}$$

Ex 8.7: Find the slope of the tangent line to the curve:

$$\boxed{x^8 + \underbrace{4x^2y^2}_{\text{product}} + y^8 = 6 \text{ at } (1,1)}$$

$$8x^7 + 4\left[(2x)(y^2) + (x^2)(2y \cdot y')\right] + 8y^7 \cdot y' = 0$$

$$y'\left[8x^2y + 8y^7\right] = -8x^7 - 8xy^2$$

$$\Rightarrow y' = -\frac{8x^7 + 8xy^2}{8y^7 + 8yx^2} = -\frac{\cancel{8}(x^7 + xy^2)}{\cancel{8}(y^7 + yx^2)}$$

$$y'\Big|_{(1,1)} = -\frac{1+1}{1+1} = -\frac{2}{2} = \boxed{-1}$$

Exercise p 60:

Find y' (using implicit differentiation):

8.2) $\underbrace{xye^x}_{\text{triple product}} + (x+2y)^2 = x$

$$\left[(1)(ye^x) + (x)(y')(e^x) + (xy)(e^x)\right] + 2[x+2y](1+2y') = 1$$

$$y'\left(xe^x + 4(x+2y)\right) = 1 - ye^x - xye^x - 2(x+2y)$$

$$\boxed{y' = \frac{1 - ye^x - xye^x - 2(x+2y)}{xe^x + 4(x+2y)}}$$

8.4 $x = y + y^{1/3} \overset{\frac{d}{dx}}{\Rightarrow} 1 = y' + \frac{2}{3}y^{-1/3}y' = y'\left[1 + \frac{2}{3\sqrt[3]{y}}\right]$

$$\Rightarrow 1 = y'\left[\frac{3\sqrt[3]{y}+2}{3\sqrt[3]{y}}\right] \Rightarrow \boxed{y' = \frac{3\sqrt[3]{y}}{3\sqrt[3]{y}+2}}$$

8.5: $\left(1+e^{-x}\right)^2 = \ln(x+y)$

$$2\left(1+e^{-x}\right)^1\left(-e^{-x}\right) = \frac{1}{x+y}\left(1+y'\right)$$

$$-2e^{-x}\left(1+e^{-x}\right) - \frac{1}{x+y} = \frac{y'}{x+y}$$

$$\boxed{y' = -2e^{-x}\left(1+e^{-x}\right)(x+y) - 1}$$

8.6: $\ln y = y^3 + \ln x$

$$\frac{1}{y}y' = (3y^2)(y') + \frac{1}{x}$$

$$y'\left[\frac{1}{y} - 3y^2\right] = \frac{1}{x}$$

$$y'\left[\frac{1-3y^3}{y}\right] = \frac{1}{x} \Rightarrow \boxed{y' = \frac{y}{x(1-3y^3)}}$$

8.11: $xy^2 = 1 + \ln(xy)$

$$\left[1 \cdot y^2 + x(2yy')\right] = \frac{1}{xy}\left[1 \cdot y + x \cdot y'\right]$$

$$y^2 + 2xyy' = \frac{1}{x} + \frac{y'}{y}$$

$$y'\left[2xy - \frac{1}{y}\right] = \frac{1}{x} - y^2$$

$$y'\left[\frac{2xy^2-1}{y}\right] = \frac{1-xy^2}{x}$$

$$\boxed{y'} = \frac{1-xy^2}{x} \cdot \frac{y}{2xy^2-1} = \boxed{\frac{y(1-xy^2)}{x(2xy^2-1)}}$$

8.12 $e^y + x^2e^x = 18$

$$e^y \cdot y' + \left[(2x)(e^x) + (x^2)(e^x)\right] = 0$$

$$y' = -\frac{e^x(2x+x^2)}{e^y}$$

$$\boxed{y' = -(2x+x^2) \cdot e^{x-y}}$$

8.13 $\underbrace{y^2 \cdot \ln y}_{\text{product}} = \underbrace{x^3 \cdot e^x}_{\text{product}}$

$$(2y \cdot y')(\ln y) + (y^2)\left[\frac{1}{y} \cdot y'\right] = 3x^2 \cdot e^x + x^3 \cdot e^x$$

$$y'\left[2y \cdot \ln y + y\right] = x^2e^x(3+x)$$

$$\boxed{y' = \frac{x^2e^x(3+x)}{y(2\ln y+1)}}$$

8.15: $\frac{2}{x} + \frac{7}{y} = 9$

$$\left(\frac{1}{u}\right)' = \frac{-1}{u^2} \cdot u'$$

$$\left(-\frac{2}{x^2}\right)(1) + \left(\frac{-7}{y^2}\right)(y') = 0$$

$$\left(-\frac{7}{y^2}\right)y' = \frac{2}{x^2} \Rightarrow \boxed{y' = -\frac{2y^2}{7x}}$$

Find y' at the indicated pt:

(Find the slope of the tg. line to the curve at the given pt.)

8.18 $3x-2y+8x^2+5y^2+9e^{9x}+7e^{2y}=16$ at $(0,0)$

$3-2y'+16x+10y.y'+81e^{9x}+14e^{2y}.y'=0$

$y'[-2+10y+14e^{2y}]=-3-16x-81e^{9x}$

$$\boxed{y'=-\frac{3+16x+81e^{9x}}{-2+10y+14e^{2y}}}$$

$$y'\Big|_{(0,0)}=-\frac{3+0+81e^{0}}{-2+0+14e^{0}}=-\frac{\cancel{84}\ \cancel{42}\ 21}{\cancel{12}\ \cancel{6}\ 3}=\boxed{-7}$$

8.22 $\sqrt{11+y^2}-12xy+2y^2+4x=0$

y' at $(1,5)=?$

$(11+y^2)^{1/2}-12xy+2y^2+4x=0$

$\frac{1}{\cancel{2}}(11+y^2)^{-1/2}\cdot(\cancel{2}y\,y')-12((1)(y)+(x)(y'))+4yy'+4=0$

$\frac{y'.y}{\sqrt{11+y^2}}-12y-12xy'+4yy'+4=0$

$y'\left[\frac{y}{\sqrt{11+y^2}}-12x+4y\right]=12y-4$

$y'\Big|_{(1,5)}\cdot\left[\frac{5}{\sqrt{11+25}}-12(1)+4(5)\right]=12(5)-4$

$y'\left[\frac{5-8(6)}{6}\right]=56 \Rightarrow y'=\frac{(56)(6)}{-43}=-\frac{336}{43}$

8.24 $\ln(xy)+xy^2-\ln(3x)-6y=0$ at $(2,3)$

$\left(\frac{1}{xy}\right)[1\cdot y+x\cdot y']+[1\cdot y^2+x\cdot 2y\cdot y']-\frac{1}{\cancel{3}x}(\cancel{3})-6y'=0$

$y'\left[\frac{\cancel{x}}{xy}+2xy-6\right]=\cancel{\frac{1}{x}}-\cancel{\frac{y}{xy}}-y^2$

$y'\left[\frac{1}{y}+2xy-6\right]=-y^2$

$y'\left(\frac{1+2xy^2-6y}{y}\right)=-y^2 \Rightarrow y'=-\frac{y^3}{1+2xy^2-6y}$

$$y'\Big|_{(2,3)}=-\frac{27}{1+2(2)(3)^2-6(3)}=\boxed{-\frac{27}{19}}$$

$1+36-18$

$37-18$