M1 contents:

Chs. 1-5 of Lecture Notes:

5.12: Let $f(x)=\begin{cases}\log\left(\frac{x}{2}+b\right), & \text{if } x<8\\ x\left(\sqrt{x-8}+\frac{1}{4}\right) & \text{if } x\geq 8\end{cases}$

Find "b" if $f(x)$ is cont. at 8?

$* f(8)=8\left(\sqrt{\cancel{8-8}}_{0}+\frac{1}{4}\right)=\boxed{2}$

$* \lim\limits_{\substack{x\to 8^+\\ x>8}} f(x)=\lim\limits_{x\to 8^+}\left[x\left(\sqrt{x-8}+\frac{1}{4}\right)\right]=8\left(\sqrt{8-8}+\frac{1}{4}\right)=\boxed{2}$

$\lim\limits_{\substack{x\to 8^-\\ (x<8)}} f(x)=\lim\limits_{x\to 8^-}\left[\log\left(\frac{x}{2}+b\right)\right]=\log\left(\frac{8}{2}+b\right)=\boxed{\log(4+b)}$

$\lim\limits_{x\to 8} f(x)=\boxed{2=\log(4+b)}$

$\log_{10}(4+b)=2 \Rightarrow 10^2=4+b \Rightarrow \boxed{b=96}$

$\Rightarrow$ When $b=96$;

$f(8)=2=\lim\limits_{x\to 8} f(x)$

$\Rightarrow f$ is cont. at $x=8$

5-22) $f(x)=\begin{cases}cx^2-2, & \text{if } x\leq 2\\ \frac{x}{c}, & \text{if } x>2\end{cases}$

Except at $x=2$; $f(x)$ is cont. everywhere (cx^2-2, $\frac{x}{c}$ $\to$ cont. everywhere)

$x=2$:

$* f(2)=c(2)^2-2=\boxed{4c-2}$

$\lim\limits_{\substack{x\to 2^-\\ (x<2)}} f(x)=\lim\limits_{x\to 2^-}(cx^2-2)=4c-2$

$\lim\limits_{\substack{x\to 2^+\\ (x>2)}} f(x)=\lim\limits_{x\to 2^+}\left(\frac{x}{c}\right)=\frac{2}{c}$

$\lim\limits_{x\to 2} f(x)=\boxed{\frac{2}{c}=4c-2}$

$\Rightarrow 4c^2-2c-2=0$

$2(2c^2-c-1)=0$

$2(2c+1)(c-1)=0$

$\boxed{c=-\frac{1}{2},\ c=1}$

$c=-\frac{1}{2}$:

$* f(2)=4\left(-\frac{1}{2}\right)-2=-4$

$* \lim\limits_{x\to 2^+} f(x)=\frac{2}{-\frac{1}{2}}=-4=\lim\limits_{x\to 2^-} f(x)$

$=4\left(-\frac{1}{2}\right)-2=-4$

$c=1$:

$f(2)=4-2=2$ ✓

$\lim\limits_{x\to 2^-} f(x)=4(1)-2=2$ ✓

$\lim\limits_{x\to 2^+} f(x)=\frac{2}{1}=2$ ✓

So for $c=-\frac{1}{2}$ or $c=1$; $f(x)$ is cont. also at $x=2 \Rightarrow$ f is cont. everywhere for $c=-\frac{1}{2}$, $c=1$

5-23) $f(x)=\begin{cases}x^2-c^2, & \text{if } x\leq 1\\ (x-c)^2, & \text{if } x>1\end{cases}$

Since f is a polynomial for $x>1$ & $x<1 \Rightarrow f$ is cont. there

$\underline{x=1:}$

$f(1) = 1^2 - c^2 = \boxed{1-c}$

$\lim\limits_{x\to 1} f(x) = ?$

$\lim\limits_{\substack{x\to 1^- \\ (x<1)}} f(x) = \lim\limits_{x\to 1^-} (x^2 - c^2) = \boxed{1-c^2}$

$\lim\limits_{\substack{x\to 1^+ \\ (x>1)}} f(x) = \lim\limits_{x\to 1^+} (x-c)^2 = (1-c)^2 = \boxed{1-2c+c^2}$

$\Rightarrow 1-c^2 = 1-2c+c^2$

$2c^2 - 2c + \cancel{1} - \cancel{1} = 0$

$2c(c-1) = 0$

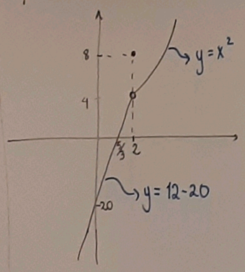
for $\boxed{c=0 \text{ or } c=1} \Rightarrow \lim\limits_{x\to 1} f(x) = 1 = f(1)$ for $c=0$

$\lim\limits_{x\to 1} f(x) = 0 = f(1)$ for $c=1$

f is cont. at $x=1$

5-20) $f(x) = \begin{cases} 12x-20, & \text{if } x<2 \\ 8, & \text{if } x=2 \\ x^2, & \text{if } x>2 \end{cases}$

Find all discontinuities of the following function and justify your answer.



* $f(2) = 8$
* $\lim\limits_{x\to 2} f(x) = ?$

$\lim\limits_{\substack{x\to 2^- \\ (x<2)}} f(x) = \lim\limits_{x\to 2^-} (12x-20) = 4$

$\lim\limits_{\substack{x\to 2^+ \\ (x>2)}} f(x) = \lim\limits_{x\to 2^+} (x^2) = 4$

$\Rightarrow \lim\limits_{x\to 2} f(x) = 4 \neq 8 = f(2) \Rightarrow$ f is not cont. at $x=2$. (discont.)

eg: $f(x) = \begin{cases} 12x-20, & \text{if } x<2 \\ \text{"a"}, & \text{if } x=2 \\ x^2, & \text{if } x>2 \end{cases}$ What should be the value of "a" so that f is cont. everywhere?

$a = f(2) = \lim\limits_{x\to 2} f(x) = 4$

$a = 4$