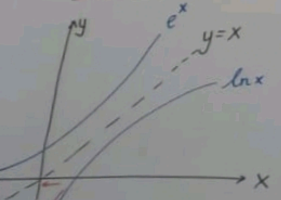
Ex 5-4:



Find the limit $\underbrace{\lim_{x\to 0^+} (\ln x)}_{-\infty} = ?$

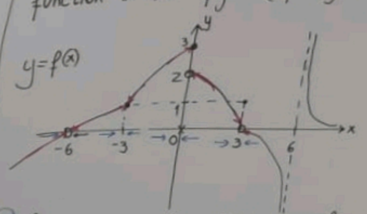
... the graph

... $x)$: this is meaningless

... domain of $\ln x$ is $(0, \infty)$.

Ex 5-5: Find the limits based on the function in the figure (if they exist?):

$y = f(x)$



a) $\lim_{\substack{x\to -6^- \\ (x<-6)}} f(x) = 0$, $\lim_{\substack{x\to -6^+ \\ (x>-6)}} f(x) = 0 \Rightarrow \lim_{x\to -6} f(x) = 0$

b) $\lim_{\substack{x\to -3^- \\ (x<-3)}} f(x) = 1$, $\lim_{\substack{x\to -3^+ \\ (x>-3)}} f(x) = 1 \Rightarrow \lim_{x\to -3} f(x) = 1$

c.) $\lim_{\substack{x\to 0^- \\ (x<0)}} f(x) = 3$, $\lim_{\substack{x\to 0^+ \\ (x>0)}} f(x) = 2 \Rightarrow \lim_{x\to 0} f(x) = \text{d.n.e.}$

(since right & left limits are different)

d.) $\lim_{\substack{x\to 3^- \\ (x<3)}} f(x) = 0$, $\lim_{\substack{x\to 3^+ \\ (x>3)}} f(x) = 0 \Rightarrow \lim_{x\to 3} f(x) = 0$

e.) $\lim_{\substack{x\to 6^- \\ (x<6)}} f(x) = -\infty$, $\lim_{\substack{x\to 6^+ \\ (x>6)}} f(x) = \infty \Rightarrow \lim_{x\to 6} f(x) = \text{d.n.e.}$

Ex 56: Evaluate the limit (if it exists):

$$\lim_{x\to 8^+} \left(\frac{x^2 - 10x + 16}{\sqrt{x-8}}\right) = \frac{0}{0}$$

$$= \lim_{x\to 8^+} \left(\frac{(x-8)(x-2)}{\sqrt{x-8}}\right) = \lim_{\substack{x\to 8^+ \\ x\neq 8}} \left(\frac{(\cancel{\sqrt{x-8}})(\sqrt{x-8})(x-2)}{\cancel{\sqrt{x-8}}}\right) = (0)(6) = \boxed{0}$$

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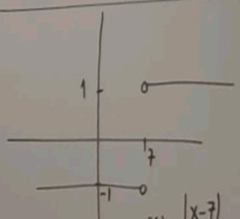
5.5) $\lim_{\substack{x\to 7^- \\ (x<7)}} \frac{|x-7|}{x-7} = \lim_{x\to 7^-} \left[\frac{-\cancel{(x-7)}}{\cancel{x-7}}\right] = \boxed{-1}$

$$|x-7| = \begin{cases} x-7, & x>7 \\ -(x-7), & x<7 \end{cases}$$

5.6) $\lim\limits_{\substack{x\to 7^+ \\ (x>7)}} \left(\frac{|x-7|}{x-7}\right) = \lim\limits_{x\to 7^+} \left(\frac{x-7}{x-7}\right) = \boxed{1}$



$f(x) = \frac{|x-7|}{x-7}$

5.8) $\lim\limits_{\substack{x\to 0^+ \\ (x>0)}} \left[\frac{\sqrt{16+3x}-4}{x}\right] = \frac{0}{0}$

$= \lim\limits_{x\to 0^+} \frac{(\sqrt{16+3x}-4)}{x} \cdot \frac{(\sqrt{16+3x}+4)}{(\sqrt{16+3x}+4)}$

$= \lim\limits_{\substack{x\to 0^+ \\ (x\neq 0)}} \frac{\cancel{16}+3\cancel{x}-\cancel{16}}{\cancel{x}(\sqrt{16+3x}+4)} = \lim\limits_{x\to 0^+} \frac{3}{(\sqrt{16+3x}+4)} = \boxed{\frac{3}{8}}$

5.11) $\lim\limits_{\substack{x\to 0^+ \\ (x>0)}} \left(\frac{2x^2+3x|x|}{x|x|}\right) = \lim\limits_{x\to 0^+} \left[\frac{2x^2+3x(x)}{x(x)}\right] = \lim\limits_{\substack{x\to 0^+ \\ (x\neq 0)}} \left(\frac{5x^2}{x^2}\right) = \boxed{5}$

5.12) $\lim\limits_{\substack{x\to 0^- \\ (x<0)}} \left(\frac{2x^2+3x|x|}{x|x|}\right) = \lim\limits_{x\to 0^-} \left(\frac{2x^2+3x(-x)}{x(-x)}\right)$

$= \lim\limits_{\substack{x\to 0^- \\ (x<0)}} \left(\frac{2x^2-3x^2}{-x^2}\right) = \lim\limits_{x\to 0^-} \frac{-x^2}{-x^2} = \boxed{1}$

Continuity:

We say that a function is continuous at $x=a$ if:

$$\boxed{\lim_{x\to a} f(x) = f(a)}$$

In other words:

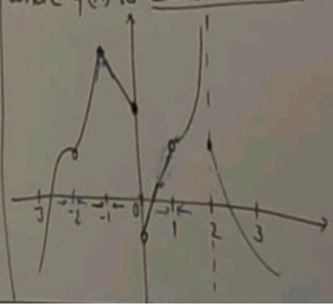
(*) f should be defined at $x=a$

(*) f should have a limit at $x=a$

(*) limit value and function value of f at $x=a$ should be equal.

If any (or all) of the above is(are) not satisfied, then we say that f is discontinuous at $x=a$

Ex 5.7 Determine the points where $f(x)$ is discontinuous.



$x=-2$: f is not defined at $x=-2$

$x=0$: limit d.n.e at $x=0$ (right & left limits are not equal)

$x=1$: f is not defined

$x=2$: f has no limit since

$\lim\limits_{x\to 2^-} f(x) = $ d.n.e (or ∞)

Ex 5-8. $f(x)=\begin{cases} 2x^2+a, & \text{if } x<2 \\ b, & \text{if } x=2 \\ 3x-2, & \text{if } x>2 \end{cases}$

Find "a" & "b" if $f(x)$ is continuous at $x=2$

* $f(2)=b$

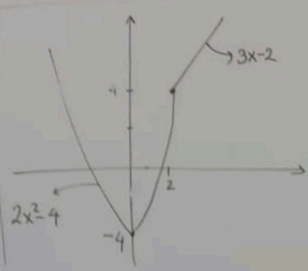
* $\lim\limits_{\substack{x\to 2^- \\ (x<2)}} f(x) = \lim\limits_{x\to 2^-}(2x^2+a) = 2(2)^2+a = \boxed{8+a}$

$\lim\limits_{\substack{x\to 2^+ \\ (x>2)}} f(x) = \lim\limits_{x\to 2^+}(3x-2) = 3(2)-2 = \boxed{4}$

$\Rightarrow \boxed{\lim\limits_{x\to 2} f(x) = \boxed{4} = 8+a} \Rightarrow \boxed{a=-4}$

* $\underbrace{f(2)}_{b} = \lim\limits_{x\to 2} f(x) = 4 \Rightarrow \boxed{b=4}$

Ans: $\boxed{\begin{matrix} a=-4 \\ b=4 \end{matrix}}$



5-21) $f(x)=\begin{cases} a+bx^2 & \text{if } x<0 \\ b & \text{if } x=0 \\ 2+e^{-x} & \text{if } x>0 \end{cases}$

* $f(0)=b$

* $\lim\limits_{\substack{x\to 0^- \\ (x<0)}} f(x) = \lim\limits_{x\to 0^-}(a+bx^2) = \boxed{a}$

$\lim\limits_{\substack{x\to 0^+ \\ (x>0)}} f(x) = \lim\limits_{x\to 0^+}(2+e^{-x}) = 2+\underbrace{e^{-0}}_{1} = \boxed{3}$

$\Rightarrow \lim\limits_{x\to 0} f(x) = \boxed{a=3}$

* $\underbrace{f(0)}_{b} = \lim\limits_{x\to 0} f(x) = 3 \Rightarrow b=3$

Ans.: $\boxed{\begin{matrix} a=3 \\ b=3 \end{matrix}}$

$\ln e = 1$

$\log 10 = 1$

$\log_a a = 1$

$\begin{pmatrix} a>0 \\ a\neq 1 \end{pmatrix}$

5.24) $f(x)=\begin{cases} e^{ax} & \text{if } x\leq 0 \\ \ln(b+x^2) & \text{if } x>0 \end{cases}$

$* \; f(0)=e^{a(0)}=e^{0}=1$ ✓

$* \; \lim\limits_{\substack{x\to 0^-\\(x<0)}} f(x)=\lim\limits_{x\to 0^-}\left(e^{ax}\right)=e^{0}=\boxed{1}$

$\lim\limits_{\substack{x\to 0^+\\(x>0)}} f(x)=\lim\limits_{\substack{x\to 0^+\\(x>0)}}\left[\ln(b+x^2)\right]=\boxed{\ln b}$

$\Big\} \; \lim\limits_{x\to 0} f(x)=1$

$\ln b=1 \Leftrightarrow \boxed{b=e}$

$* \; f(0)=1=\lim\limits_{x\to 0} f(x)$ when $b=e$

and "a" is an arbitrary real number

5.9) $\lim\limits_{\substack{x\to -2^+\\(x>-2)\\(x<2)}} \dfrac{|x^2-4|}{x+2}=\lim\limits_{x\to -2^+}\dfrac{-(x-2)\cancel{(x+2)}}{\cancel{(x+2)}}=+4$

$\lim\limits_{\substack{x\to -2^-\\(x<-2)\\(x<2)}} \dfrac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}}=-2-2=-4$

$|x^2-4|=|x-2||x+2|=\begin{cases} (-(x-2))(-(x+2)) & x<-2 \\ (x-2)(x+2), & x<-2 \\ -(x-2)(x+2) & x>-2 \end{cases}$