Lines

l_1

m_{l_1} slope of l_1

l_2

m_{l_2} slope of l_2

* $l_1 \parallel l_2$ l_1 & l_2 are parallel lines
  iff $m_{l_1} = m_{l_2}$

* $l_1 \perp l_2$ l_1 & l_2 are perpendicular (or orthogonal)
  iff $m_1 \cdot m_2 = -1$ ($\underline{\text{or}}$ $m_1 = -\frac{1}{m_2}$)

Special lines: $x = c \rightarrow$ vertical line (line $\parallel$ to y-axis)

$y = c \rightarrow$ horizontal line (line $\parallel$ to x-axis)

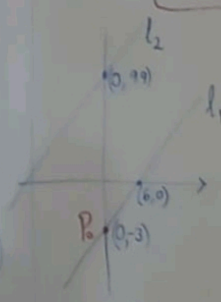
1-34) l_1 passes through $(0,-3)$ [i.e. y-intercept of l_1 is -3]
and parallel to l_2: $\underbrace{10y - 5x = 99}$

$$10y = 5x + 99$$

$$y = \left(\frac{1}{2}\right)x + \frac{99}{10}$$

$$m_{l_2} = \frac{1}{2}$$

$l_1 \parallel l_2 \Rightarrow \boxed{m_{l_1} = m_{l_2}}$



$$m_{l_1} = \frac{1}{2}$$

$$P_0(0,-3)$$

l_1: $y - (-3) = \left(\frac{1}{2}\right)(x - 0)$

$$y = \frac{1}{2}x - 3 \Rightarrow 2y - x = -6$$

$$\boxed{x - 2y = 6}$$

1-35) l_1: $\boxed{(9,12)} \in l_1$

$l_1 \perp l_2$ $\underbrace{2x + 5y = 60} \Rightarrow 5y = -2x + 60$

$$y = \left(-\frac{2}{5}\right)x + 12$$

$$\boxed{m_{l_2} = -\frac{2}{5}}$$

$$\boxed{m_1} = -\frac{1}{m_2} = -\frac{1}{\left(-\frac{2}{5}\right)} = \boxed{\frac{5}{2}}$$

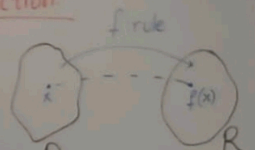
l_1: $y - 12 = \left(\frac{5}{2}\right)(x - 9)$

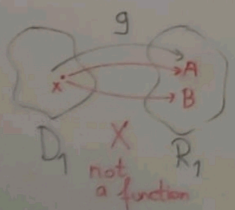
$$2y - 24 = 5x - 45$$

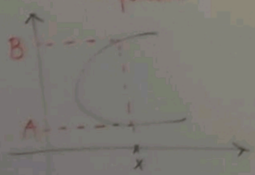
$$45 - 24 = 5x - 2y$$

l_1: $\boxed{5x - 2y = 21}$

Function







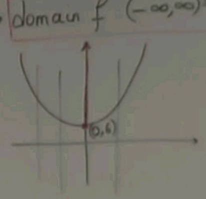
(Dom $f \subseteq$ x-axis)

Domain The set of all x values for which $f(x)$ is defined is the domain of $f(x)$

e.g. i) $f(x) = x^2 + 6 \rightarrow \boxed{\text{domain } f \ (-\infty, \infty) = \mathbb{R}}$

$x^2 \geq 0$, $\underbrace{x^2+6}_{y} \geq 6$

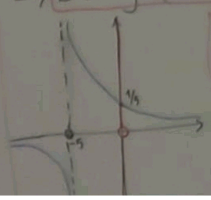
Range f: $[6, \infty)$



ii) $g(x) = \dfrac{1}{x+5} \Rightarrow g$ is defined $\forall\, x \neq -5$

$\Rightarrow \boxed{\text{Dom } g: (-\infty, -5) \cup (-5, \infty)}$

$\boxed{\text{Range } (-\infty, 0) \cup (0, \infty)}$



Range Set of all $f(x)$ values (for $x \in$ Dom f) is called the range of f

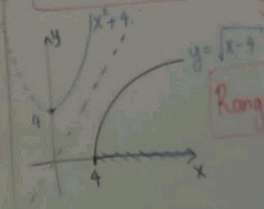
(ie range $f \subseteq$ y-axis)

$\hookrightarrow$ subset

eg $f(x) = \sqrt{x-4}$

f is defined $\forall\, x$ values s.t. $x \geq 4$

$\boxed{\text{Dom } f \ [4, \infty)}$ $\left[\ \Rightarrow x-4 \geq 0 \quad y = \sqrt{x-4} \geq 0 \right]$



$\boxed{\text{Range } [0, \infty)}$

eg $g(x) = \dfrac{1}{\sqrt{x-4}}$

$\boxed{\text{Dom } g(x) = (4, \infty)}$

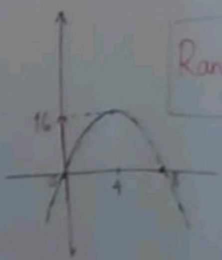
$\boxed{\text{Range } g(x) = (0, \infty)}$

Find the domain and range of the following functions

1-38) $f(x) = 8x - x^2 = x(8-x)$

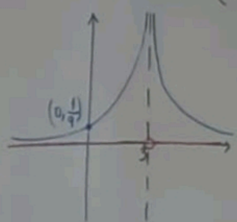
$\boxed{\text{Dom } f = (-\infty, \infty) \text{ (or } \mathbb{R})}$

$f(x) = -(x^2 - 8x) = -[(x-4)^2 - 16] = \boxed{-(x-4)^2 + 16}$



$\boxed{\text{Range } f \ (-\infty, 16]}$

1-39) $f(x) = \frac{1}{x^2 - 6x + 9} = \frac{1}{(x-3)^2}$



Dom f $(-\infty, 3) \cup (3, \infty)$

Range f $(0, \infty)$

Ch.2 Parabolas

Quadratic Equations

(*) $\boxed{ax^2 + bx + c = 0}$ quadratic eqn

(**) $\boxed{x_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}}$ quadratic formula

$a \neq 0$, $\Delta = b^2 - 4ac$ (discriminant)

Solution(s) of (*) eqn is(are) determined according to the sign of Δ:

- $\Delta > 0 \Rightarrow$ 2 distinct real solns of (*)
- $\Delta = 0 \Rightarrow$ 2 repeated (identical) real roots

  $x_1 = x_2 = -\frac{b}{2a}$
- $\Delta < 0 \Rightarrow$ no real solns. (roots) of (*)

Ex 2-2: Solve $8x^2 - 6x - 5 = 0$

$a = 8,\ b = -6,\ c = -5$

$\Delta = b^2 - 4ac = (-6)^2 - 4(8)(-5)$
$= 36 + 160 = 196$

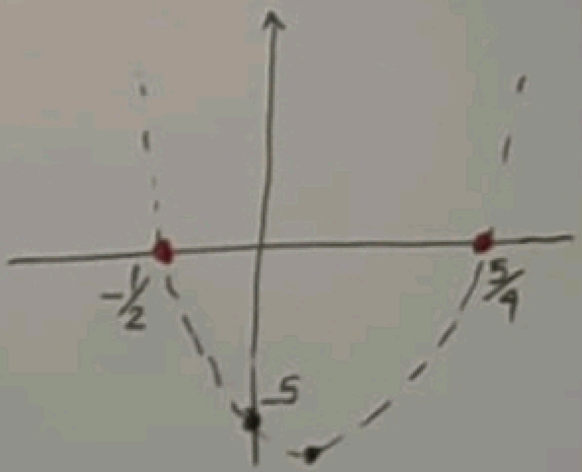
$x_{1,2} = \frac{-(-6) \mp \sqrt{196}}{2(8)} = \frac{6 \mp 14}{16}$ → $\boxed{\frac{5}{4}}$, $\boxed{-\frac{1}{2}}$

$\boxed{x_1 = \frac{5}{4},\ x_2 = -\frac{1}{2}}$

$(x - x_1)(x - x_2) = 0$

$(x - \frac{5}{4})(x + \frac{1}{2}) = 0$

$\boxed{(4x-5)(2x+1) = 0}$



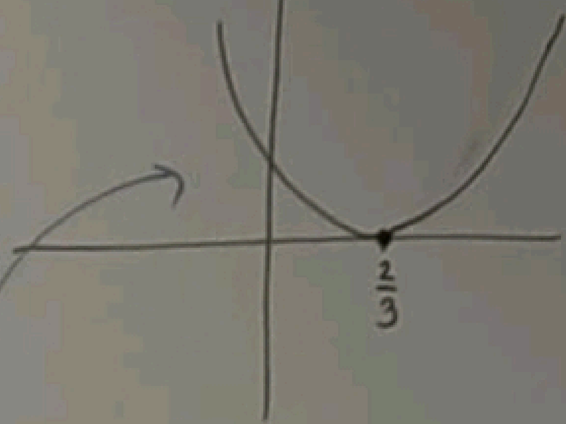
Ex. 2-3 $\boxed{9x^2-12x+4=0}$

$$\Delta = b^2-4ac = (-12)^2-4(9)(4)$$
$$= 144-144$$
$$= \boxed{0}$$

$\Rightarrow$ repeated real soln.

$$x_1 = x_2 = -\frac{b}{2a} = -\frac{(-12)}{2(9)} = \boxed{\frac{2}{3}}$$

$$\Rightarrow \left(x-\frac{2}{3}\right)^2 = 0$$
$$(3x-2)^2 = 9x^2-12x+4=0$$



Ex 2-4: $\boxed{3x^2+6x+4=0}$

$$\Delta = (6)^2-4(3)(4) = 36-48$$
$$= -12 < 0$$

$\Rightarrow$ no real solns of the given quadratic eqn.