Quadratic eqn: $ax^2+bx+c=0$

Quadratic func.: $\underbrace{f(x)=ax^2+bx+c}_{\text{parabola}}$, $a\neq 0$

The graph of the quadratic function $f(x)$ has shape according to:

* if $a>0 \Rightarrow$ parabola opens upward $\cup$
* if $a<0 \Rightarrow$ 〃 〃 downward $\cap$

Vertex of the parabola is either the maximum or the minimum pt.

$$\boxed{\text{Vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)}$$

$$f(x)=a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right]$$

$$f(x)=a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+\frac{c}{a}\right]$$

$$f(x)=a\left(x+\frac{b}{2a}\right)^2+\left(c-\frac{b^2}{4a}\right)$$

$$\Rightarrow \boxed{V\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)}$$

Ex 2-5: Sketch the graph of:

$f(x)=x^2-10x+16$

$a=1>0 \Rightarrow \cup$ ✓

$a=1,\ b=-10,\ c=16$

Intercepts:

y-intercept: $x=0 \Rightarrow y=16$

$\boxed{(0,16)}$ ✓

x-intercept(s) (if any): $\boxed{(2,0),(8,0)}$ ✓ ✓

$x^2-10x+16=0$

$(x-2)(x-8)=0 \Rightarrow x=2,\ x=8$

$-\frac{b}{2a}=-\frac{(-10)}{2(1)}=\boxed{5}$

$\Rightarrow f(5)=25-50+16=\boxed{-9}$

$\boxed{\text{Vertex: } (5,-9)}$ ✓

$f(x)=(x-5)^2-25+16=\boxed{(x-5)^2-9}$

✓

Find the vertex, and x- and y-intercepts (if any) of the following parabolas. Sketch their graphs.

2-12) $y=\underbrace{-x^2+12}_{f(x)}$ $\quad a=-1<0\ \cap,\ b=0,\ c=12$

Vertex: $-\frac{b}{2a}=0 \Rightarrow f(0)=12 \Rightarrow \boxed{(0,12)}$ Vertex

y-intercept: $x=0 \Rightarrow y=12 \Rightarrow \boxed{(0,12)}$ y-int.

x-intercept(s): $-x^2+12=0 \Rightarrow x^2=12$ $\quad \boxed{x=\mp 2\sqrt{3}}$

$\boxed{(2\sqrt{3},0),(-2\sqrt{3},0)}$

Range:

$(-\infty,12]$

2-15) $f(x) = y = x^2 + 10x + 25$ $a=1>0$ ∪ ✓

$= (x+5)^2 + 0$

y-intercept: $x=0 \Rightarrow y=25$

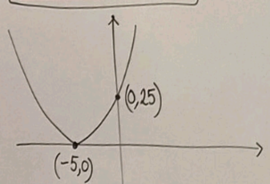
$(0,25)$ ✓

X-intercept(s): $(x+5)^2 = 0$

$\Rightarrow x_1 = x_2 = -5$

✓ $\boxed{(-5,0)} \rightarrow$ double root

$\boxed{\text{Vertex: } (-5,0)}$



2-19) $y = f(x) = x^2 - 4x + 5$ $a=1>0$ ∪

y-intercept: $(0,5)$ ✓

X-intercept(s): $x^2 - 4x + 5 = 0$

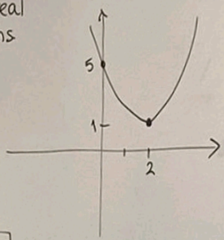
$\Delta = (-4)^2 - 4(1)(5) < 0 \Rightarrow$ no-real solns

$\Rightarrow$ $\boxed{\text{no x-intercepts}}$ ✓

$\boxed{\text{Vertex}}$: $\left(-\frac{b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$= \left(-\frac{-4}{2(1)}, 1\right) = \boxed{(2,1)}$ ✓

$f(x) = (x-2)^2 - 4 + 5 = (x-2)^2 + 1$



2-20) $y = -3x^2 + 60x - 450$, $a=-3<0 \Rightarrow$ ∩ ✓

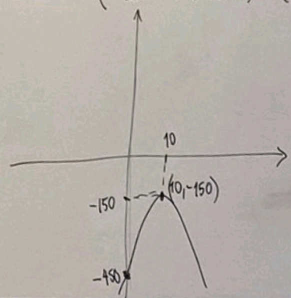
y-intercept: $x=0 \Rightarrow y=-450$ $\boxed{(0,-450)}$ ✓

X-intercept(s): $\Delta = (60)^2 - 4(-3)(-450) = 3600 - 5400 < 0$

$\Rightarrow$ no real roots of the quadratic function

$\Rightarrow$ no x-intercepts of the parabola.

Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \left(-\frac{60}{2(-3)}, f(10)\right) = (10,-150)$



Ch.3:

Polynomials: $n \in \mathbb{Z}_+$

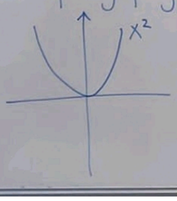
$$\boxed{p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}$$

is called a polynomial of degree n.

eg. $x^5 - 3x^2 + 7 \rightarrow$ polyn. of degree 5

$x^{-2}, \sqrt{x}, \dfrac{1}{x-5}, x^{7/4}, \ldots \rightarrow$ not polynomials

Domain of any polynomial function is $\mathbb{R}$.



Rational functions:

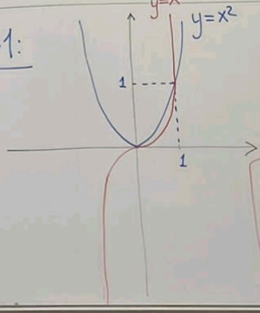
The quotient of two polynomials determine rational funcs.

$$r(x) = \frac{p(x)}{q(x)} \; \text{polynomials}$$

Domain of $r(x)$ is all x's for which $q(x) \neq 0$

eg. $\dfrac{3x^2 - 5}{1 + 2x - 7x^3}$ rational func.

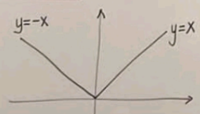
Ex. 3.1:



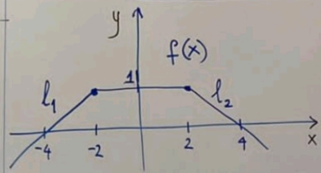
$$\boxed{\begin{array}{l} 0 < x < 1 \\ x^3 < x^2 \end{array}}$$

Piece-wise defined function:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

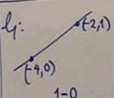


Ex. 3-2: Find the formula of the function $f(x)$:



$$f(x) = \begin{cases} -\frac{1}{2}x + 2 & x > 2 \\ 1, & -2 \leq x \leq 2 \\ \frac{1}{2}x + 2, & x < -2 \end{cases}$$

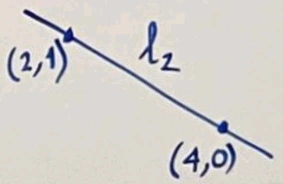
l_1:



$m_1 = \dfrac{1-0}{-2-(-4)} = \dfrac{1}{2}$

$y - 0 = \left(\frac{1}{2}\right)(x - (-4))$

$$\boxed{y = \frac{1}{2}x + 2}$$



$$m_2 = \frac{1-0}{2-4} = -\frac{1}{2}$$

$$y - 0 = \left(-\frac{1}{2}\right)(x-4)$$

$$\boxed{y = -\frac{1}{2}x + 2}$$

3-2) Sketch the graph of the following function:

$$f(x) = \begin{cases} x+3, & x<4 \\ x-1, & x \geq 4 \end{cases}$$