Functions:

Equations:

Solve: $8x-2=5x+7$

$8x-5x=7+2$

$\frac{3x}{3}=\frac{9}{3} \Rightarrow \boxed{x=3}$

Soln: $\{3\}$

Ex. 1-1: Solve the eqn.: $\frac{2x}{2x+5}=\frac{3}{4} \Longrightarrow$

$4(2x)=3(2x+5)$

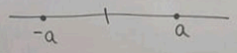
$8x=6x+15$

$2x=15 \Rightarrow \boxed{x=\frac{15}{2}}$

Soln.: $\left\{\frac{15}{2}\right\}$

Absolute value:

$|x|=a \Rightarrow \begin{matrix} x=a \\ \text{or} \\ x=-a \end{matrix}$



Ex. 1-2: Solve the eqn.: $|3x-12|=27$

$\Rightarrow 3x-12=27$ or $3x-12=-27$

$-(3x-12)=27$

$\Downarrow$

$3x=39$ $\qquad$ $3x=-27+12$

$\boxed{x=13}$ or $3x=-15$

$\boxed{x=-5}$

Soln.: $\{-5,13\}$

Intervals:

Closed interval: $[a,b]=\{x \mid a \le x \le b\}$

Open interval: $(a,b)=\{x \mid a<x<b\}$

Half-open interval: $(a,b]=\{x \mid a<x \le b\}$

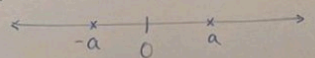
$[a,b)=\{x \mid a \le x<b\}$

Unbounded intervals:

$(a,\infty)=\{x \mid x>a\}$ $\left(\text{or } [a,\infty)=\{x \mid x \ge a\}\right)$

$(-\infty,a)=\{x \mid x<a\}$ $\left(\text{or } (-\infty,a]=\{x \mid x \le a\}\right)$

$\mathbb{R} \rightarrow$ real numbers : $(-\infty,\infty)$



Inequalities:

Ex. 1-3: Solve the inequality:

$7x-5 \le 30$

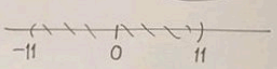
$\frac{7x}{7} \le \frac{30+5}{7}=\frac{35}{7}$

$x \le 5 \Rightarrow$ **Soln.:** $(-\infty,5]$

Ex1-4: $|x+10|<11$

$x+10<11$ or $-(x+10)<11$

$x+10>-11$



$x+10<11$ or $x+10>-11$

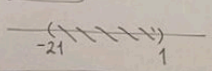
$\Downarrow$

$x<11-10=1$

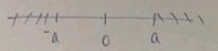
$x>-11-10$

$x<1$

$x>-21$



Soln.: $(-21,1)$



$|x|>a \Rightarrow x>a$ or $x<-a$

Ex1-5: $|x+10|>11$

$x+10>11$ or $x+10<-11$

$(-(x+10)>11$

$x+10<-11)$

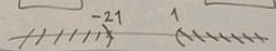
$\Downarrow$

$x>11-10=1$

$x<-11-10$

$x>1$

$x<-21$



$(-\infty,-21)\cup(1,\infty)$

1-28) $|12-7x|\geq 1$

$12-7x\geq 1$ or $-(12-7x)\geq 1$

$12-7x\leq -1$

$\Downarrow$

$12-1\geq 7x$

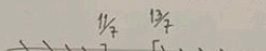
$\frac{11}{7}\geq x$

$\Downarrow$

$13\leq 7x$

$x\geq \frac{13}{7}$

$\frac{13}{7}\leq x$



Soln.: $(-\infty,\frac{11}{7}]\cup[\frac{13}{7},\infty)$

p9

1-2) $\left(\frac{1}{16}\right)^{3/4}=?$

$(a)^{m/n}=\sqrt[n]{a^m}=(\sqrt[n]{a})^m$

$=\frac{1^{3/4}}{16^{3/4}}=\frac{1}{\left((2^4)^{1/4}\right)^3}=\frac{1}{2^3}=\frac{1}{8}$

1-5) $\sqrt[3]{\frac{8}{1000}}=\frac{(2^3)^{1/3}}{(10^3)^{1/3}}=\frac{2}{10}=\frac{1}{5}$

1-12) $\sqrt{x^3y}\cdot\sqrt{64xy^9}$

$=\sqrt{64x^4y^{10}}=\sqrt{64}\cdot(x^4)^{1/2}\cdot(y^{10})^{1/2}$

$=8x^2y^5$

1

1 2 1

1 3 3 1

1 4 6 4 1

1-7) $(a+b)^2 = 1a^2 + 2ab + 1b^2$

$\boxed{(a-b)^2 = (a+(-b))^2 = a^2 - 2ab + b^2}$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a+(-b))^3$

$a^2 - b^2 = (a-b)(a+b)$

1-9) $\frac{1}{\sqrt{5}-\sqrt{3}} \left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right) = \frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{\sqrt{5}+\sqrt{3}}{5-3} = \frac{\sqrt{5}+\sqrt{3}}{2} = \boxed{\frac{\sqrt{5}}{2}+\frac{\sqrt{3}}{2}}$

1-10) $\frac{12}{\sqrt{7}-1} - \frac{12}{\sqrt{7}+1} = \frac{12(\sqrt{7}+1) - 12(\sqrt{7}-1)}{(\sqrt{7})^2-(1)^2}$

$(\sqrt{7}+1)$ $(\sqrt{7}-1)$ $7-1=6$

$= \frac{12\sqrt{7}+12-12\sqrt{7}+12}{6} = \frac{24}{6} = \boxed{4}$

$\boxed{a^3 - b^3 = (a-b)(a^2+ab+b^2)}$

$a^3 - b^3 \mid a-b$

$-a^3 + a^2b \quad a^2+ab+b^2$

$a^2b - b^3$

$-a^2b + ab^2$

$ab^2 - b^3$

$-ab^2 + b^3$

0

$a^3 + b^3 = a^3 - (-b)^3$

$= (a+b)(a^2 - ab + b^2)$

1-13) $x^3 - 1 = (x-1)(x^2+x+1)$

1-14) $\left(\sqrt{x^2+4}+3\right)\left(\sqrt{x^2+4}-3\right) =$

$\left(\sqrt{x^2+4}\right)^2 - (3)^2 = (x^2+4) - 9 = \boxed{x^2-5}$

1-15) $x^4 - 100y^4 = ?$

$(x^2)^2 - (10y^2)^2 =$

$(x^2-10y^2)(x^2+10y^2)$

1-17) $(3a-2b)^2 = ?$

$= (3a)^2 - 2(3a)(2b) + (2b)^2$

$= \boxed{9a^2 - 12ab + 4b^2}$

1-19) $\frac{2x}{x^2-4} + \frac{5}{x+2} = ?$

$(x-2)(x+2)$ $(x-2)$

$= \frac{2x + 5(x-2)}{(x-2)(x+2)}$

$= \boxed{\frac{7x-10}{x^2-4}}$

1-20) $1 - \frac{1}{1+\frac{1}{x}} = ?$

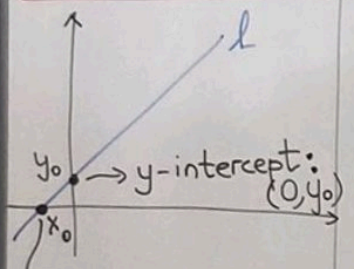
$= 1 - \frac{1}{\frac{x+1}{x}} = 1 - \frac{x}{x+1}$

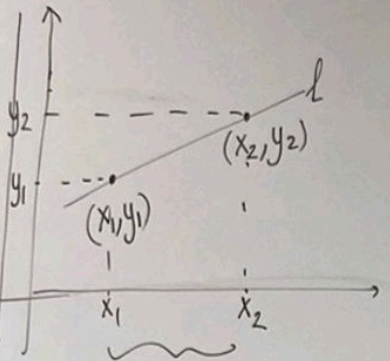
$= \frac{(x+1)-x}{x+1} = \boxed{\frac{1}{x+1}}$

$$\frac{a}{\frac{b}{c}} = a \cdot \frac{c}{b}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$$

Lines in the Plane





$$\Delta x = x_2 - x_1$$

$$\Delta y = y_2 - y_1$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

m ↓ slope of ℓ

* slope-intercept form of eqn. of ℓ:

$$y - y_1 = m(x - x_1) \Rightarrow y = mx + \underbrace{(y_1 - mx_1)}_{n}$$

$$\boxed{y = mx + n}$$

$x = 0 \Rightarrow y = n \Rightarrow (0,n)$: y-intercept of ℓ

* point-slope form of ℓ:

$$y - y_1 = m(x - x_1)$$

1-31) Find the eqn. of the line that passes through the origin (i.e. $(0,0)$) and has slope $m = \frac{1}{5}$

$$y - 0 = \frac{1}{5}(x - 0) \Rightarrow y = \frac{1}{5}x \Rightarrow 5y = x$$

$$\Rightarrow \boxed{x - 5y = 0}$$