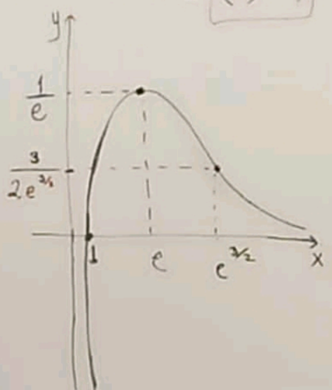
8.7 $\boxed{f(x) = \dfrac{\ln x}{x}}$

$$\text{Dom } f \cdot \{x \mid x \neq 0,\ x > 0\}$$
$$= \boxed{(0, \infty)}$$



$$\lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \frac{1}{\infty} = \boxed{0} \leftarrow y=0 \text{ as } x \to \infty$$

$$\lim_{x\to 0^+} \left(\frac{\ln x}{x}\right) = \frac{-\infty}{0} \ ?$$

$$= \lim_{x\to 0^+} \left[\left(\frac{1}{x}\right)(\ln x)\right] = (\infty)(-\infty) = \boxed{-\infty}$$

Intercepts:

$x > 0 \Rightarrow \boxed{\text{no-y-int}}$.

$$y = 0 \Rightarrow \frac{\ln x}{x} = 0 \Rightarrow \ln x = 0$$
$$\Rightarrow \boxed{x = 1}$$
$$\Rightarrow \boxed{\text{x-int: } (1,0)}$$

$$f'(x) = \frac{\left(\frac{1}{\cancel{x}}\right)(\cancel{x}) - (1)(\ln x)}{x^2} = \boxed{\frac{1 - \ln x}{x^2}} = \begin{cases} = 0 \Rightarrow \boxed{\ln x = 1} \Rightarrow \boxed{x = e} \\ \cancel{= \text{d.n.e.} \Rightarrow x = 0} \end{cases}$$

not in the domain

Crit pt $(e, f(e)) = \left(e, \frac{\ln e}{e}\right)$

$$\Rightarrow \boxed{\left(e, \frac{1}{e}\right)}$$

x	0	1	e	e^2
f'		+		−
f		↗		↘

local max pt

f incr on: $(0, e)$

$f \searrow$ on (e, ∞)

$$f''(x) = \frac{\left(-\frac{1}{x}\right)(x^2) - (2x)(1 - \ln x)}{(x^2)^2}$$

$$= \frac{-x - 2x + (2x)(\ln x)}{x^4}$$

$$= \frac{\cancel{x}(2\ln x - 3)}{x^{\cancel{4}3}} = \boxed{\frac{2\ln x - 3}{x^3}}$$

$$\boxed{f''(x) = \frac{2\ln x - 3}{x^3}} = \begin{cases} 0 \Rightarrow \ln x = \frac{3}{2} \Rightarrow x = e^{3/2} \\ \text{dne} \Rightarrow \cancel{x = 0} \text{ (not in the domain)} \end{cases}$$

Inf pt : $\left(e^{3/2}, f(e^{3/2})\right) = \left(e^{3/2}, \frac{\frac{3}{2}}{e^{3/2}}\right)$

x	0	1	$e^{3/2}$	e^2
f''		$-$	•	$+$
f		$\cap$		$\cup$

f concave down: $(0, e^{3/2})$

f 〃 up: $(e^{3/2}, \infty)$

local max. pt : $(0,0)$

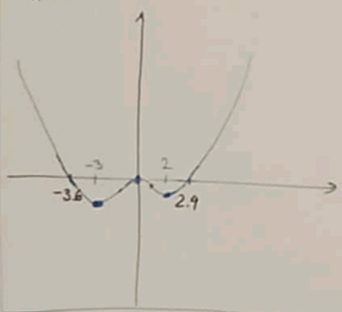
local min pts. : $(-3, f(-3))$, $(2, f(2))$

$\Leftarrow$

8-8 $\boxed{f(x) = 5x^6 + 6x^5 - 45x^4}$

Dom $f(x) = (-\infty, \infty)$

$\lim_{x \to \mp\infty} f(x) = \infty$



x	-4	-3	-1	0	1	2	3
f'	$-$	•	$+$	•	$-$	•	$+$
f	↘		↗		↘		↗

Intercepts:

$x = 0 \Rightarrow f(0) = 0 \Rightarrow$ y-int: $(0,0)$

$f(x) = 0 \Rightarrow x^4(5x^2 + 6x - 45) = 0 \Rightarrow x_{1,2,3,4} = 0$

quadruple root

$$x_{5,6} = \frac{-6 \mp \sqrt{36 - 4(5)(-45)}}{2 \times 5} \cong \frac{-6 \mp 30.59}{10} \cong \begin{cases} -3.6 \\ 2.4 \end{cases}$$

x - ints.

$$f'(x) = 30x^5 + 30x^4 - 180x^3$$
$$= 30x^3(x^2 + x - 6) = 0$$
$$= \boxed{30x^3 (x+3)(x-2)}$$

$\Rightarrow$ Crit (pts): $x = 0, x = -3, x = 2$

$(0, f(0)), (-3, f(-3)), (2, f(2))$

$f \nearrow$: $(-3, 0) \cup (2, \infty)$

$f \searrow$: $(-\infty, -3) \cup (0, 2)$

Intercepts:

$x=0 \Rightarrow f(0)=0 \Rightarrow$ y-int: $(0,0)$

$f(x)=0 \Rightarrow \underbrace{x^4 \underbrace{(5x^2+6x-45)}_{0}}=0 \Rightarrow x_{1,2,3,4}=0$

quadruple root

$$x_{5,6}=\frac{-6\mp\sqrt{36-4(5)(-45)}}{2\times 5} \cong \frac{-6\mp 30.59}{10} \cong \begin{cases} -3.6 \\ 2.4 \end{cases}$$

x-ints.

$$f'(x)=30x^5+30x^4-180x^3$$
$$=30x^3(x^2+x-6)=0$$
$$=\boxed{30x^3\cdot(x+3)(x-2)}$$

$\Rightarrow$ Crit (pts): $x=0, x=-3, x=2$

$(0,f(0)), (-3,f(-3)), (2,f(2))$

$f \nearrow : (-3,0) \cup (2,\infty)$

$f \searrow : (-\infty,-3) \cup (0,2)$

$$f''(x)=150x^4+120x^3-540x^2$$
$$=\underbrace{30x^2}_{0}\underbrace{(5x^2+4x-18)}_{0}=0$$

$x_{1,2}=0$

$$\Rightarrow x_{3,4}=\frac{-4\mp\sqrt{16-4(5)(-18)}}{10}$$
$$=\frac{-4\mp\sqrt{376}}{10} \cong \begin{cases} 1.5 \to \alpha \\ -2.3 \to \beta \end{cases}$$

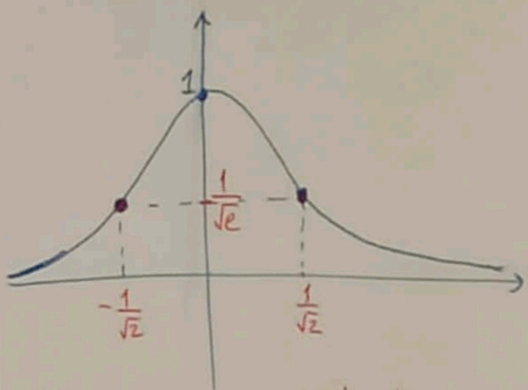
Concavity:

x	-3	β	-1	0	1	α	2
f''	$+$	$\bullet$	$-$	$\bullet$	$-$	$\bullet$	$+$
f	$\cup$		$\cap$		$\cap$		$\cup$

8.17 $f(x)=e^{-x^2}$

Dom f: $(-\infty,\infty)$

$\lim_{x\to\mp\infty} e^{-(\mp\infty)^2} = e^{-\infty} = \frac{1}{\infty} = 0$



Gaussian distribution

* $e^{-x^2}\neq 0 \Rightarrow \boxed{\text{No x-int}}$

* y-int: $x=0 \Rightarrow e^{-0}=1 \Rightarrow \boxed{(0,1)\ \text{y-int}}$

$f' = \underbrace{e^{-x^2}\cdot(-2x)}_{\frac{-2x}{e^{x^2}}} = 0 \Rightarrow x=0$

~~d.n.e~~ no x-value

Crit pt: $(0,f(0))=(0,1)$

x	-1	0	1
$f'=\frac{-2x}{e^{x^2}}$	+		−
f	↗		↘

f ↗ on: $(-\infty,0)$

f ↘ on: $(0,\infty)$

local max pt: $(0,1)$

$f''(x)=(-2)\,e^{-x^2}+(-2x)\left(e^{-x^2}(-2x)\right) = -2\underbrace{e^{-x^2}}_{\neq 0}\underbrace{\left[1-2x^2\right]}_{=0}=0$

$f''(x)=0 \Leftrightarrow x=\mp\frac{1}{\sqrt{2}} \Rightarrow$ inf pts $\left(\mp\frac{1}{\sqrt{2}}, f\left(\mp\frac{1}{\sqrt{2}}\right)\right)=\left(\mp\frac{1}{\sqrt{2}}, e^{-\frac{1}{2}}\right)=\left(\mp\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{e}}\right)$

x	-1	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	
f''	+		−		+
f	∪		∩		∪

f ∩ $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

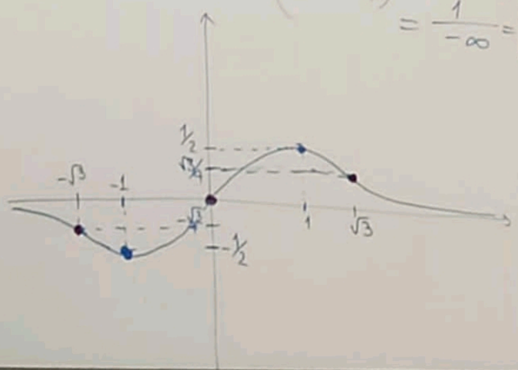
f ∪ $\left(-\infty,-\frac{1}{\sqrt{2}}\right)\cup\left(\frac{1}{\sqrt{2}},\infty\right)$

818 $\boxed{f(x)=\frac{x}{x^2+1}}$

Dom f: $(-\infty,\infty)$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\cancel{x}}{\cancel{x}(x+\frac{1}{x})} = \frac{1}{\infty+0} = \boxed{0^+}$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\left(\frac{x}{x(x+\frac{1}{x})}\right) = \frac{1}{-\infty+0} = \frac{1}{-\infty} = \boxed{0^-}$$



Intercepts:

$x=0 \Rightarrow y=\frac{0}{1}=0 \Rightarrow (0,0)\to$ y-int

$y=0 \Rightarrow \frac{x}{x^2+1}=0 \iff x=0 \Rightarrow (0,0)\to$ x-int.

$$f'(x)=\frac{(1)(x^2+1)-(2x)(x)}{(x^2+1)^2}=\frac{1-x^2}{(x^2+1)^2}=0 \iff \boxed{x=\mp 1}$$

Crit pts:

$(1,f(1))=(1,\frac{1}{2}) \to$ local max pt

$(-1,f(-1))=(-1,-\frac{1}{2}) \to$ local min pt

x	−2	−1	0	1	2
f'	−		+		−
f	↘	local min pt	↗	local max pt	↘

$$f'=\frac{1-x^2}{(x^2+1)^2}$$

$$f''=\frac{(-2x)(x^2+1)^2-2(x^2+1)(2x)(1-x^2)}{(x^2+1)^4}$$

$$=\frac{-2x\cancel{(x^2+1)}\left[(x^2+1)+2(1-x^2)\right]}{(x^2+1)^{\cancel{4}\,3}}$$

$$=\frac{(-2x)(3-x^2)}{(x^2+1)^3}=\begin{cases} =0 \Rightarrow \boxed{x=0 \ \& \ x=\mp\sqrt{3}} \\ \cancel{=\text{dne}} \ (\text{no such pt}) \end{cases}$$

x	−2	$-\sqrt{3}$	−1	0	1	$\sqrt{3}$	2
f''	−		+		−		+
f	∩		∪		∩		∪