Chain rule:

If f and g are differentiable, then $f(g(x))$ is also differentiable and

$$\big(f(g(x))\big)' = f'(g(x)) \cdot g'(x)$$

$$\left(\begin{array}{l} y = f(u) \\ u = g(x) \end{array} \right)$$

or more briefly;

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$y \rightarrow u \rightarrow x$

Ex 6-8: Find $\frac{d}{dx}(3x^2+1)^5 = \frac{dy}{dx} = ?$

$u = 3x^2+1$, $y = f(u) = u^5$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (5u^4)(6x) = 5(3x^2+1)^4 \cdot (6x)$$

$u = 3x^2+1$

$$= (30x)(3x^2+1)^4$$

Ex 6-9: $f(x) = e^{x^5} \Rightarrow f'(x) = ?$

$u = x^5$, $y = f(u) = e^u$ } $f(u(x)) = e^{x^5}$

$$f'(x) = e^{x^5} \cdot (5x^4) = 5x^4 \cdot e^{x^5}$$

Use chain rule to evaluate the derivatives of the following functions:

6-27) $f(x) = \ln(1+x^2)$ $\left[\begin{array}{l} u = 1+x^2 \\ f(u) = \ln u \end{array} \right]$

$$f'(x) = \frac{1}{1+x^2} \cdot (2x) = \frac{2x}{1+x^2}$$

$f(x) = a^{u(x)}$ $\left(\begin{array}{l} a \neq 1 \\ a > 0 \end{array} \right)$

$$f'(x) = a^{u(x)} \cdot u'(x)$$

6-28) $f(x) = \left(\underbrace{5+x+2x^3}_{u}\right)^7$

$f'(x) = 7\left(5+x+2x^3\right)^6 \cdot \left(1+6x^2\right)$

$= \left(7+42x^2\right)\left(5+x+2x^3\right)^6$

6-29) $f(x) = \frac{x}{\sqrt{3x^2+2}} = \frac{x}{\left(3x^2+2\right)^{1/2}}$

$f'(x) = \frac{(1)\left(3x^2+2\right)^{1/2} - x \cdot \left[\frac{1}{2}\left(3x^2+2\right)^{-1/2} \cdot (6x)\right]}{\left(\sqrt{3x^2+2}\right)^2}$

$= \frac{\sqrt{3x^2+2} - \frac{3x^2}{\sqrt{3x^2+2}}}{\left(3x^2+2\right)^1}$

$= \frac{\left(3x^2+2\right) - 3x^2}{\left(3x^2+2\right)^{3/2}} = \frac{2}{\left(\sqrt{3x^2+2}\right)^3}$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$, $\frac{d}{dx}\left(\ln(u(x))\right) = \frac{u'(x)}{u(x)}$, $\frac{d}{dx}\left(\log_a(u(x))\right) = \left(\frac{\ln u(x)}{\ln a}\right)'$

$= \frac{1}{\ln a} \cdot \frac{u'(x)}{u(x)}$

6-33) $f(x) = \sqrt{1+\ln x} = \left(1+\ln x\right)^{1/2} \Rightarrow f'(x) = ?$

$f'(x) = \frac{1}{2}\left(1+\ln x\right)^{-1/2} \cdot \left(\frac{1}{x}\right) = \frac{1}{2x\sqrt{1+\ln x}}$

6-34) $f(x) = \sqrt{x^2+2e^{3x}} \Rightarrow f'(x) = ?$

$= \left(x^2+2e^{3x}\right)^{1/2}$

$f'(x) = \frac{1}{2}\left(x^2+2e^{3x}\right)^{-1/2} \cdot \left(2x+2e^{3x} \cdot (3)\right)$

$= \frac{2\left(x+3e^{3x}\right)}{2\sqrt{x^2+2e^{3x}}} = \frac{x+3e^{3x}}{\sqrt{x^2+2e^{3x}}}$

6.35) $f(x)=4^{x^2+5x} \Rightarrow f'(x)=4^{x^2+5x}\cdot(2x+5)$

6.36) $f(x)=(xe^x)(\log_3(x+x^4))$

$$f'(x)=(e^x+xe^x)(\log_3(x+x^4)) + (xe^x)\left(\frac{1}{\ln 3}\cdot\frac{(1+4x^3)}{(x+x^4)}\right)$$

$$=(e^x+xe^x)(\log_3(x+x^4))+\frac{(xe^x)(1+4x^3)}{(\ln 3)(x+x^4)}$$

Find $f''=?$

6.37) $f(x)=5^{2x}$

$f'(x)=5^{2x}\cdot(2)$

$f''(x)=2\left[5^{2x}\cdot 2\right]=4\cdot 5^{2x}$

$f^{(n)}(x)=2^n\cdot 5^{2x}$

6.38) $f(x)=\ln(3x)$

$f'(x)=\frac{1}{3x}\cdot(3)=\frac{1}{x}=x^{-1}$

$f''(x)=-x^{-2}=\frac{-1}{x^2}$

6.39) $f(x)=\sqrt{2+x}=(2+x)^{1/2}$

$f'(x)=\frac{1}{2}(2+x)^{-1/2}\cdot(1)=\frac{1}{2\sqrt{2+x}}$

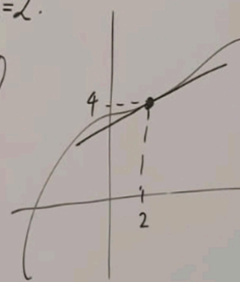
$f''(x)=\frac{1}{2}\left[-\frac{1}{2}(2+x)^{-3/2}\right](1)$

$=\frac{-1}{4\sqrt{(2+x)^3}}$

6.46) Find the eqn. of the tangent line to $f(x)=x^2(1-x)^2$ at $x=2$.

$f(2)=2^2(1-2)^2=4(-1)^2=4$

pt: $(2,4)$



m= slope $=f'(2)=?$

$f'(x)=(2x)(1-x)^2+x^2[2(1-x)(-1)]$

$f'(x)=2x(1-x)^2-2x^2(1-x)$

$f'(2)=4(-1)^2-2(4)(1-2)=4+8=12=m$

tg. line eqn: $y-4=m(x-2)$

tg. line eqn: $y-4=12(x-2) \Rightarrow y-4=12x-24$

$y=12x-20$