14.) $\lim\limits_{x\to5}\left(\dfrac{\sqrt{x-1}-2}{\underbrace{x^2-6x+5}_{(x-5)(x-1)}}\right) \underset{\left(\frac{0}{0}\right)}{=} \lim\limits_{\substack{x\to5\\x\neq5}}\left[\dfrac{\overbrace{(\sqrt{x-1}-2)(\sqrt{x-1}+2)}^{\cancel{x-5}}}{\cancel{(x-5)}(x-1)(\sqrt{x-1}+2)}\right] = \lim\limits_{x\to5}\left[\dfrac{1}{(x-1)(\sqrt{x-1}+2)}\right]$

conjugate
= eşlenik

$= \dfrac{1}{4\cdot4} = \dfrac{1}{16}$

$\underset{a-b}{(\sqrt{x-1}-2)}\underset{a+b}{(\sqrt{x-1}+2)} = a^2-b^2 = x-1-4 = x-5$

15.) $\lim\limits_{x\to2}\left(\dfrac{\sqrt{3x+10}-4}{x-2}\right) \underset{\left(\frac{0}{0}\right)}{=} \cdots = \dfrac{3}{8}$

45) $\lim\limits_{x\to3}\left(\dfrac{x^4-81}{x^3-27}\right) = \left(\dfrac{0}{0}\right)$

$x^4-81 = [(x^2)^2-(9)^2] = (x^2-9)(x^2+9) = (x-3)(x+3)(x^2+9)$

$x^3-(3)^3 \quad (x-3)(x^2+3x+9)$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$= \lim\limits_{\substack{x\to3\\x\neq3}}\left[\dfrac{\cancel{(x-3)}(x+3)(x^2+9)}{\cancel{(x-3)}(x^2+3x+9)}\right]$

$= \dfrac{(6)(9+9)}{(9+9+9)} = \dfrac{12}{3} = 4$

46) $\lim\limits_{x\to1}\left(\dfrac{x^n-1}{x-1}\right) = ?$

$x^2-1 = (x-1)\boxed{(x+1)}$

$x^3-1 = (x-1)\boxed{(x^2+x+1)}$

$x^4-1 = (x-1)(x+1)(x^2+1) = (x-1)\boxed{\overbrace{(x^3+x^2+x+1)}^{4\text{-terms}}}$

$x^5-1 = (x-1)\boxed{\underbrace{(x^4+x^3+x^2+x+1)}_{5\text{-terms}}}$

$$\begin{array}{r|l} x^5-1 & x-1 \\ \hline -x^5 \pm x^4 & x^4+x^3+x^2+x+1 \\ x^4-1 & \\ -x^4 \pm x^3 & \\ x^3-1 & \\ -x^3 \pm x^2 & \\ x^2-1 & \\ -x^2 \pm x & \\ x-1 & \\ x-1 & \\ 0\ 0 & \end{array}$$

$x^n-1 = (x-1)\underbrace{(x^{n-1}+x^{n-2}+\cdots\cdots+x+1)}_{n\text{-terms}}$

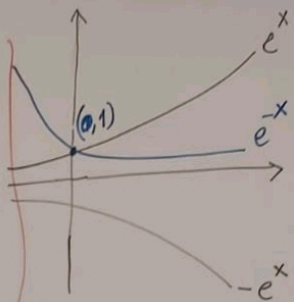
$= \lim\limits_{\substack{x\to1\\x\neq1}}\dfrac{\cancel{(x-1)}(x^{n-1}+x^{n-2}+\cdots+x+1)}{\cancel{(x-1)}}$

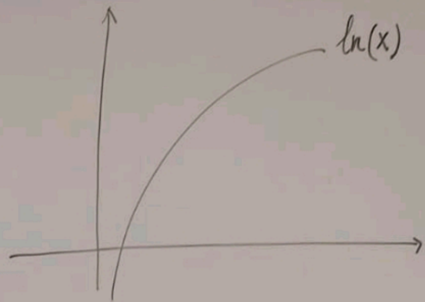
$= \overbrace{1+1+\cdots+1}^{n\text{-terms}} = n$

ii) $\lim_{x\to-\infty}\left(\frac{1+x+3x^4}{x+x^2}\right) = \lim_{x\to-\infty}\left[\left(3x^2-3x+3\right)+\underbrace{\left(\frac{-2x+1}{x^2+x}\right)}_{0}\right]$

$= (-\infty)^2 + 0 = \infty$

$$
\begin{array}{l|l}
3x^4+x+1 & x^2+x \\ \hline
-3x^4 \mp 3x^3 & 3x^2-3x+3 \\
\hline
-3x^3+x+1 & \\
\pm 3x^3 \mp 3x^2 & \\
\hline
3x^2+x+1 & \\
-3x^2 \mp 3x & \\
\hline
-2x+1 &
\end{array}
$$





7) $\lim_{x\to\infty} e^x = e^\infty = \infty$, $\lim_{x\to-\infty}(e^x) = e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0$

8) a) $\lim_{x\to\infty}\left(e^{-x}+3\right) = \frac{1}{e^\infty}+3 = \frac{1}{\infty}+3 = 0+3$

b) $\lim_{x\to-\infty}\left(e^{-x}+2x+5\right) = e^{-(-\infty)}+(-\infty)+5$

$= \infty + (-\infty) = \infty$

Since $e^{-(-\infty)} \to \infty$ much faster than $2x$ would go to $(-\infty)$

9) $\lim_{x\to\infty}(\ln x) = \infty$

10) a) $\lim_{x\to\infty}\left(-e^{-x^3}+9\right) = -e^{-\infty}+9 = -\frac{1}{e^\infty}+9 = 0+9 = 9$

b) $\lim_{x\to-\infty}\left(-e^{-x^3}+9\right) = -e^{-(-\infty)^3}+9 = -e^{-(-\infty)}+9 = -e^\infty+9 = -\infty$

Examples: Find the following limits

(1) $$\lim_{x\to\infty} \frac{\overbrace{1+x-2x^2}^{p(x)}}{\underbrace{3x^3+2x-5}_{q(x)}} = ?$$

deg $p(x) = 2$

deg $q(x) = 2$

$$= \lim_{x\to\infty}\left[\frac{\cancel{x^2}\left(\frac{1}{x^2}+\frac{1}{x}-2\right)}{\cancel{x^2}\left(3+\frac{2}{x}-\frac{5}{x^2}\right)}\right] = \boxed{\frac{-2}{3}}$$

$\frac{2}{x} \to 0$, $\frac{5}{x^2} \to 0$ as $x \to \infty$

(2) $$\lim_{x\to\infty} \frac{2+\sqrt{x}+x^5}{\sqrt[3]{x}+1+2x^3}$$

$\to$ deg $p(x)=5$

$\to$ deg $q(x)=3$

$= \infty$ [since deg $p(x)=5 >$ deg $q(x)=3$]

(3) $$\underbrace{\lim_{x\to\infty}\left(\frac{x^3+2x+\sqrt{x}}{2x^4+3x+5}\right)}_{=\,0} = ?$$

$\leftarrow$ deg $p(x)=3$

$\leftarrow$ deg $q(x)=4$

0 (since deg $p(x) <$ deg $q(x)$)

(4) $$\lim_{x\to-\infty}\left(\frac{\overbrace{x+9x^2}^{p(x)}}{\underbrace{3x^2+1}_{q(x)}}\right) = \frac{9}{3} = 3$$

since deg $p(x) = 2 =$ deg $q(x)$

(5) $$\lim_{x\to-\infty}\left(\frac{1+x+x^2}{x+x^3}\right)$$

$\to$ deg $p(x)=2$

$\to$ deg $q(x)=3$ ^

$= 0$ (since deg $p(x) <$ deg $q(x)$)

(6) $$\lim_{x\to-\infty}\left(\frac{\overbrace{1+x+3x^3}^{p(x)}}{\underbrace{x+x^2}_{q(x)}}\right) = -\infty$$

deg $p(x) = 3 >$ deg $q(x) = 2$ and the difference of degrees is 1 } so the limit goes to $\boxed{-\infty}$

i) $3x^3+x+1 \;|\; x^2+x$

$-3x^3 \mp 3x^2$ $\qquad 3x-3$

$-3x^2+x+1$

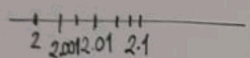
$\pm 3x^2 \pm 3x$

$4x+1$

$$\lim_{x\to-\infty}\left(\frac{3x^3+x+1}{x^2+x}\right) = \lim_{x\to-\infty}\left[\underbrace{(3x-3)}_{\to\,-\infty} + \underbrace{\frac{4x+1}{x^2+x}}_{\to\,0}\right] \to -\infty$$

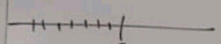
Ex: Evaluate the following limits, if they exist.

1.) $\lim_{x\to2^+}\left[\frac{1}{x-2}\right] = \frac{1}{0^+} = \infty$

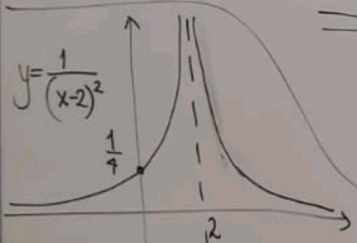


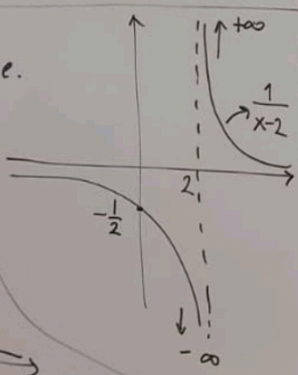
$2.1-2=0.1>0$
$2.01-2=0.01>0$
$2.001-2=0.001>0$

2.) $\lim_{x\to2^-}\left(\frac{1}{x-2}\right) = \frac{1}{0^-} = -\infty$



3.) $\lim_{x\to2}\left(\frac{1}{x-2}\right)$ = d.n.e.

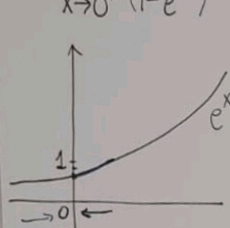




$\lim_{x\to2}\frac{1}{(x-2)^2}$ = D.N.E.

4.) $\lim_{x\to0^+}\left(\frac{e^x}{1-e^x}\right) = \frac{1}{0^+} = \infty$

(d.n.e)



$\underbrace{e^x>1}_{1-e^x<0}$, $(x>0)$ $(x\to0^+)$

$\lim_{x\to0^-}\left(\frac{e^x}{1-e^x}\right) = \frac{1}{0^+} = \infty$

(d.n.e)

$x<0 \Rightarrow e^x<1$

$\Rightarrow 1-e^x>0$

4.3) $\lim_{x\to5}\frac{x^2-25}{x^2-10x+25} = \lim_{x\to5}\frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x-5)}$

$\downarrow$ $\left(\frac{0}{0}\right)$

$= \frac{10}{0} =$ d.n.e.

Limits at Infinity:

1) $p>0 \Rightarrow \lim_{x\to\infty}\left(\frac{1}{x^p}\right)=0$, $\lim_{x\to-\infty}\left(\frac{1}{x^p}\right)=0$

[equiv.: $p<0$ $\lim_{x\to\infty}(x^p)=0$]

Limits of rational func. at $\mp\infty$: $\lim_{x\to\mp\infty}\frac{p(x)}{q(x)} = ?$

2) if [deg p(x) < deg q(x)] $\Rightarrow \lim_{x\to\mp\infty}\frac{p(x)}{q(x)} = 0$

3) if [deg p(x) = deg q(x)] $\Rightarrow \lim_{x\to\mp\infty}\frac{p(x)}{q(x)} =$ ratio of leading term coefficients

4) if [deg p(x) > deg q(x)] $\Rightarrow \lim_{x\to\mp\infty}\frac{p(x)}{q(x)} = \mp\infty$

4-9) $\lim\limits_{x\to 64}\left(\dfrac{\sqrt{x}-8}{x-64}\right) \underset{\downarrow\ \left(\frac{0}{0}\right)}{=} \lim\limits_{x\to 64}\dfrac{\cancel{(\sqrt{x}-8)}}{\cancel{(\sqrt{x}-8)}(\sqrt{x}+8)} = \dfrac{1}{8+8} = \dfrac{1}{16}$

$x-64=(\sqrt{x})^2-(8)^2$
$=(\sqrt{x}-8)(\sqrt{x}+8)$

4-10) $\lim\limits_{x\to 0}\left(\dfrac{\sqrt{2x+1}-3}{x}\right) = \dfrac{-2}{0} = \text{d.n.e.}$ $(-\infty)$

4-11) $\lim\limits_{x\to 7}\left[\dfrac{\sqrt{4x+8}-6}{x-7}\right] \underset{\downarrow\ \left(\frac{0}{0}\right)}{=} \lim\limits_{x\to 7}\left[\dfrac{(\sqrt{4x+8}-6)(\sqrt{4x+8}+6)}{(x-7)(\sqrt{4x+8}+6)}\right]$

$= \lim\limits_{\substack{x\to 7\\ x\neq 7}}\left[\dfrac{(4x-28)}{(x-7)(\sqrt{4x+8}+6)}\right] = \lim\limits_{\substack{x\to 7\\ x\neq 7}}\left[\dfrac{4\cancel{(x-7)}}{\cancel{(x-7)}(\sqrt{4x+8}+6)}\right]$

$= \dfrac{4}{12} = \dfrac{1}{3}$

Infinite Limits & Limits at Infinity:

* If f increases (or decreases) without bounds as $x\to a$, we say that the limit is infinity (or −infinity)

$\Rightarrow \lim\limits_{x\to a} f(x)=\infty$ or $\lim\limits_{x\to a} f(x)=-\infty$

* ∞ (or $-\infty$) is not a number.
If limit is ∞ (or $-\infty$), it means lim. d.n.e.